

# Why Functional Programming Matters



John Hughes & Mary Sheeran

QuviQ

**CHALMERS**

# Functional Programming à la 1940s

- Minimalist: who needs booleans?
- A boolean just *makes a choice!*

**true**    **x y = x**

**false** **x y = y**

- We can *define* if-then-else!

**ifte** **bool t e =**  
**bool t e**

# Who needs integers?

- A (positive) integer just *counts loop iterations!*

**two** **f** **x** = **f** (**f** **x**)

**one** **f** **x** = **f** **x**

**zero** **f** **x** = **x**

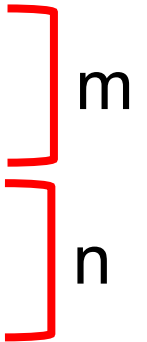
- To recover a "normal" integer...

```
*Church> two (+1) 0  
2
```

# Look, Ma, we can add!

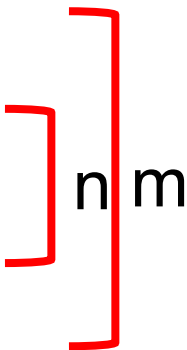
- Addition by *sequencing* loops

$$\text{add } m \ n \ f \ x = m \ f \ (n \ f \ x)$$



- Multiplication by *nesting* loops!

$$\text{mul } m \ n \ f \ x = m \ (n \ f) \ x$$



```
*Church> add one (mul two two) (+1) 0  
5
```

# Factorial à la 1940

```
fact n =  
  ifte (iszero n)  
    one  
    (mul n (fact (decr n)))
```

```
*Church> fact (add one (mul two two)) (+1) 0  
120
```

# A couple more auxiliaries

- Testing for zero

```
iszero n =  
  n (\_ -> false) true
```

- Decrementing...

```
decr n =  
  n (\m f x-> f (m incr zero))  
  zero  
  (\x->x)  
  zero
```

Booleans, integers, (and other data structures) *can be entirely replaced by functions!*

*"Church encodings"*

Early versions of the Glasgow Haskell compiler actually implemented data-structures this way!



*Alonzo Church*



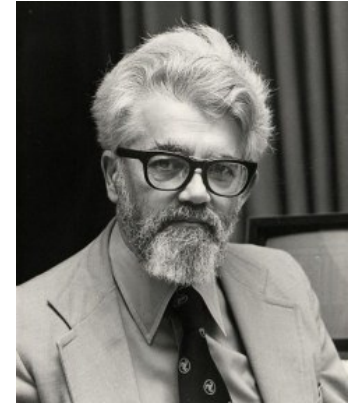




The type-checker needs a *little bit* of help

```
fact ::  
  (forall a. (a->a) ->a->a) ->  
  (a->a) -> a -> a
```

# Factorial à la 1960



```
(LABEL FACT (LAMBDA (N)
  (COND ((ZEROP N) 1)
        (T (TIMES N (FACT (SUB1 N)))))))
```

## Higher-order functions!

```
(MAPLIST FACT (QUOTE (1 2 3 4 5)))
```

```
(1 2 6 24 120)
```

# The Next 700 Programming Languages

P. J. Landin

*Univac Division of Sperry Rand Corp., New York, New York*

“... today ... 1,700 special programming languages used to ‘communicate’ in over 700 application areas.”—*Computer Software Issues*, an American Mathematical Association Prospectus, July 1965.



## Factorial in ISWIM

`fac(5)`

where `rec fac(n) =`

`(n=1) → 1;`

`n*fac(n-1)`

# Laws

(MAPLIST F (REVERSE L))  $\equiv$  (REVERSE (MAPLIST F L))

What's the point of two different ways to do the same thing?

Wouldn't *two* facilities be better than one?

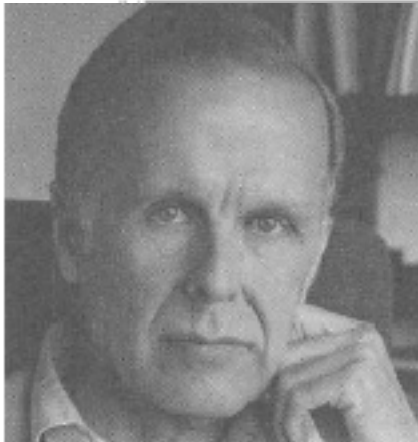
**Expressive power should be by design, rather than by accident!**



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# Can Programming Be Liberated from the von Neumann Style? A Functional Style and Its Algebra of Programs

John Backus  
IBM Research Laboratory, San Jose



Turing award 1977

[Paper 1978](#)

**Conventional programming  
languages are growing ever  
more enormous,  
but not stronger.**

**Inherent defects at the most  
basic level cause them to be  
both *fat* and *weak*:**



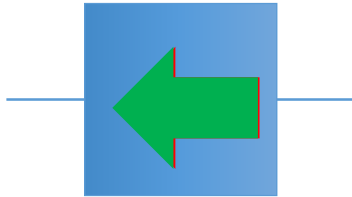
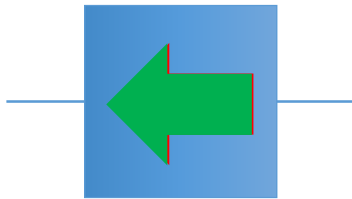
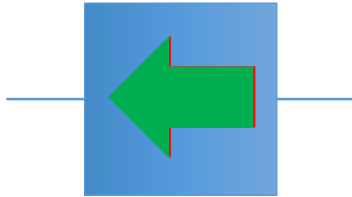
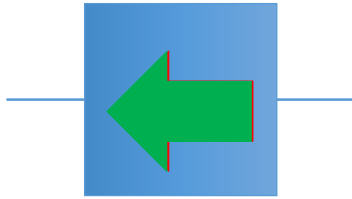
Word-at-a-time



their inability to effectively use  
powerful combining forms  
for building new programs from  
existing ones



apply to all



$\alpha f$

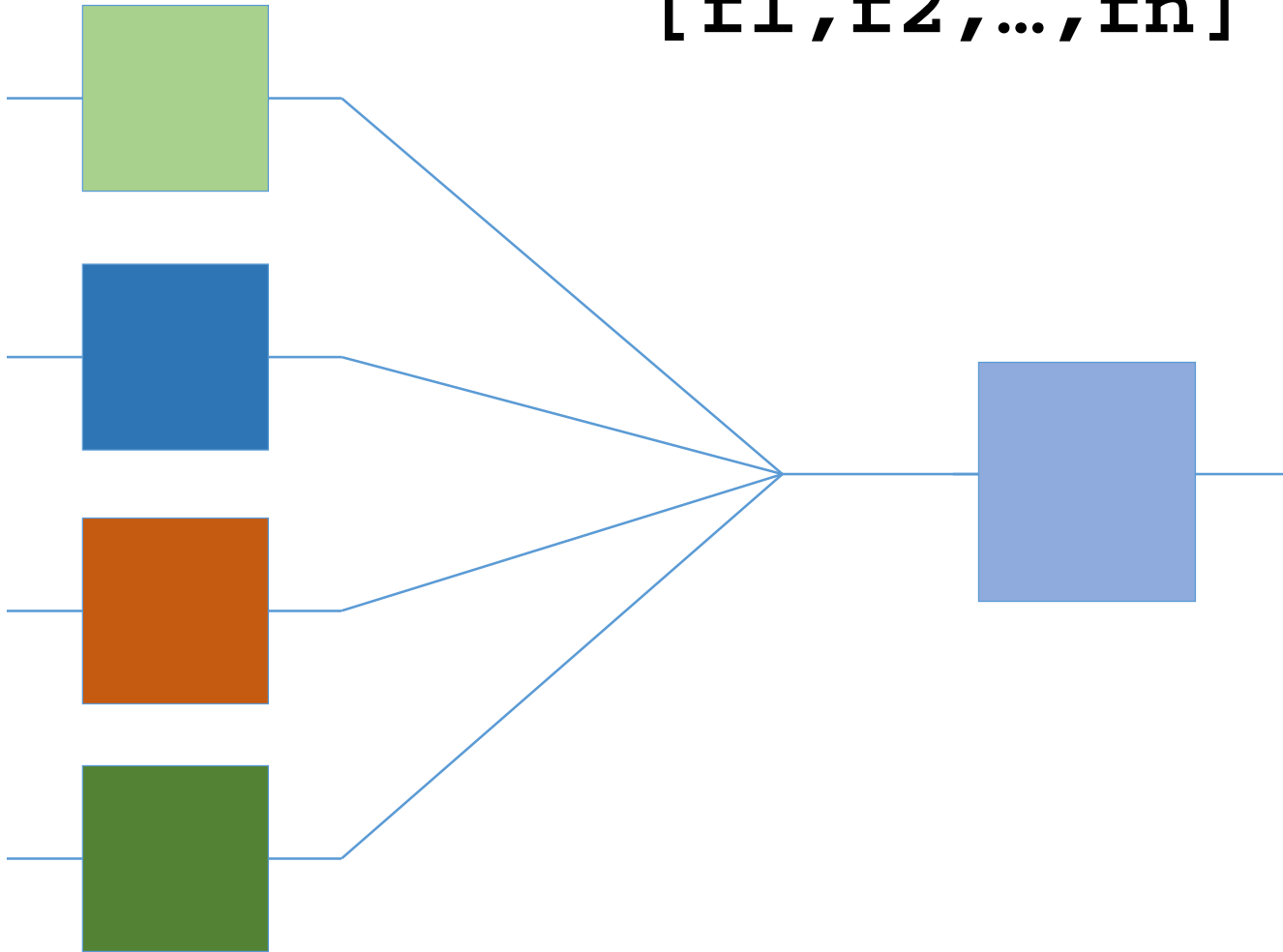
construction

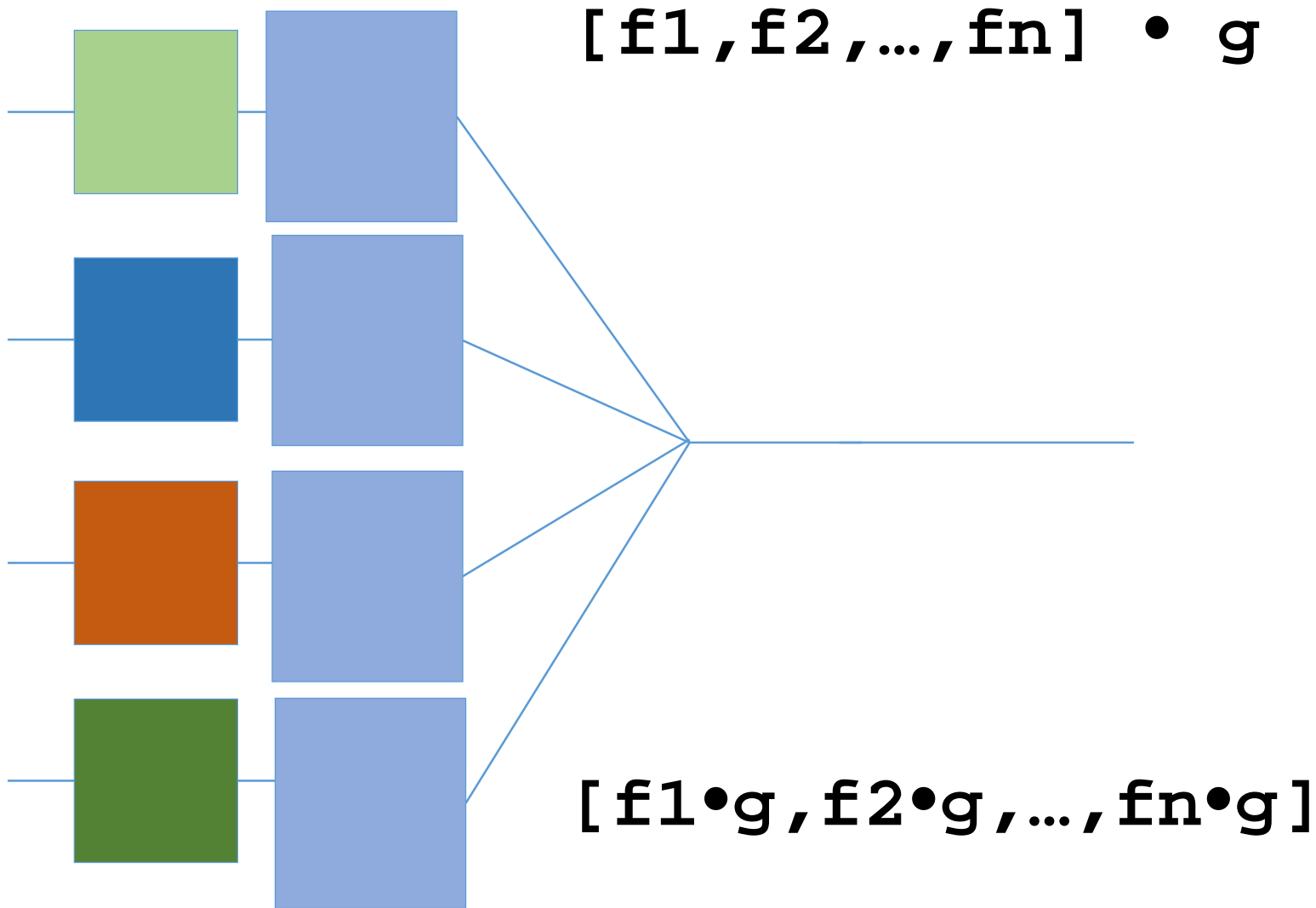


$[f_1, f_2, f_3, f_4]$

**their lack of useful  
mathematical properties for  
reasoning about programs**

$[f_1, f_2, \dots, f_n] \cdot g$



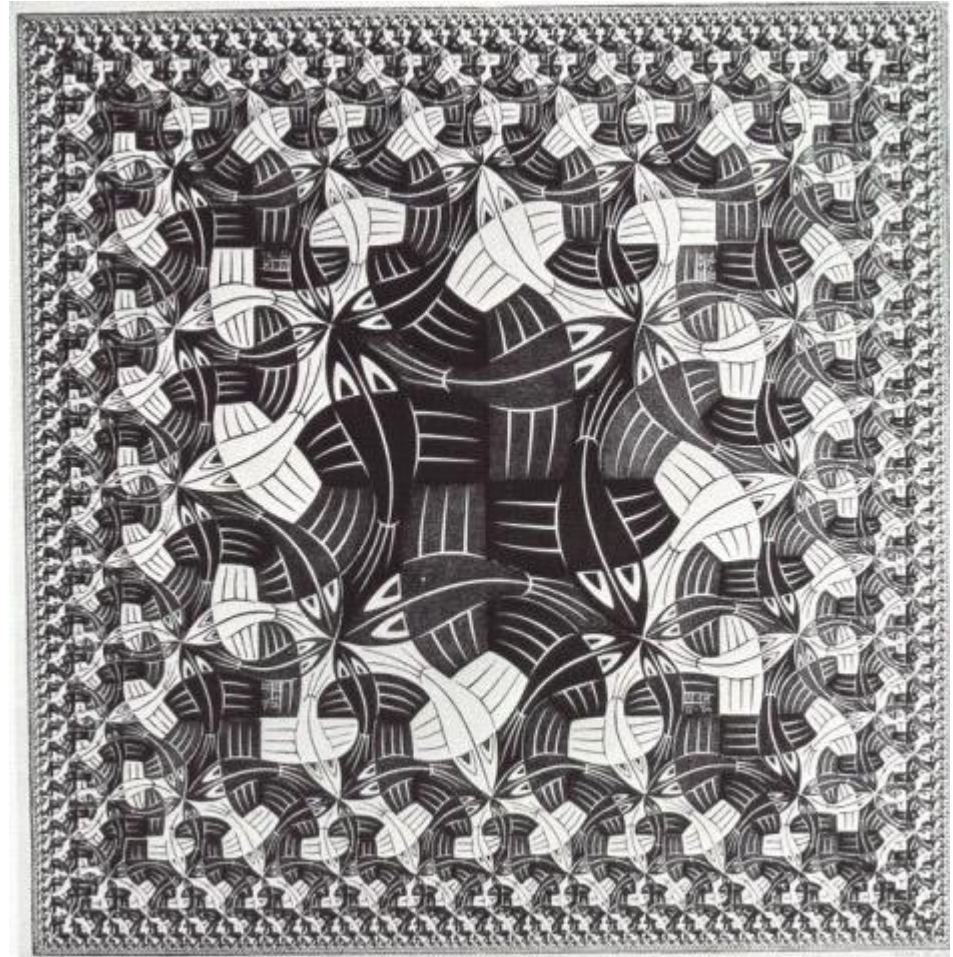


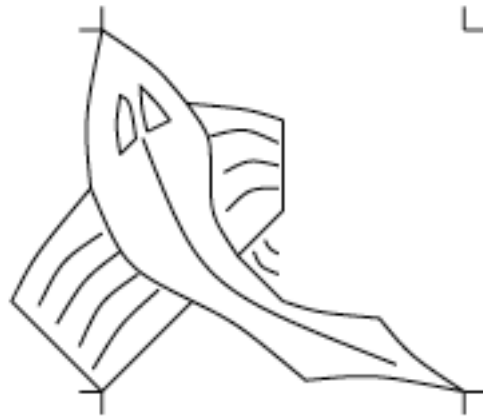


```
c := 0;  
for i := 1 step 1 until n do  
    c := c + a[i] × b[i]
```

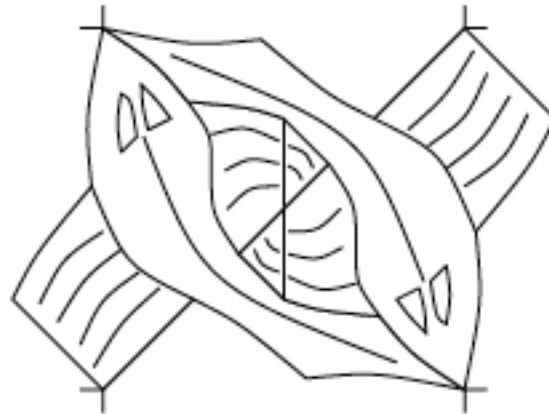
**Def IP = (/ +) • (α ×) • Trans**

*Peter Henderson, Functional Geometry, 1982*

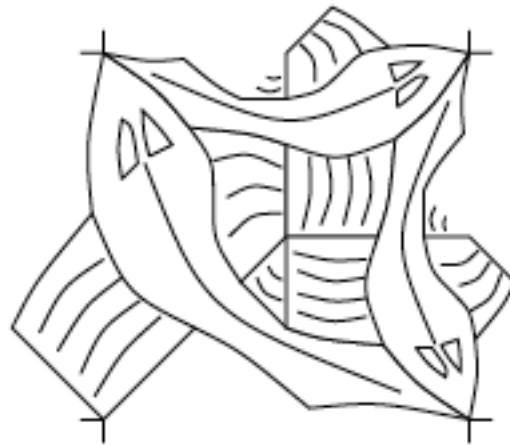




**fish**



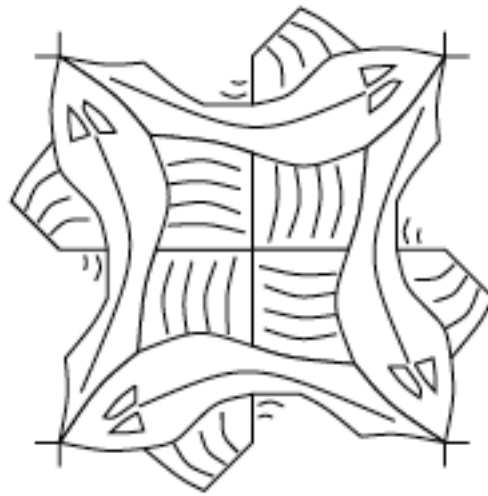
`over (fish, rot (rot (fish)))`



```
t = over (fish, over (fish2, fish3))
```

```
fish2 = flip (rot45 fish)
```

```
fish3 = rot (rot (rot (fish2)))
```

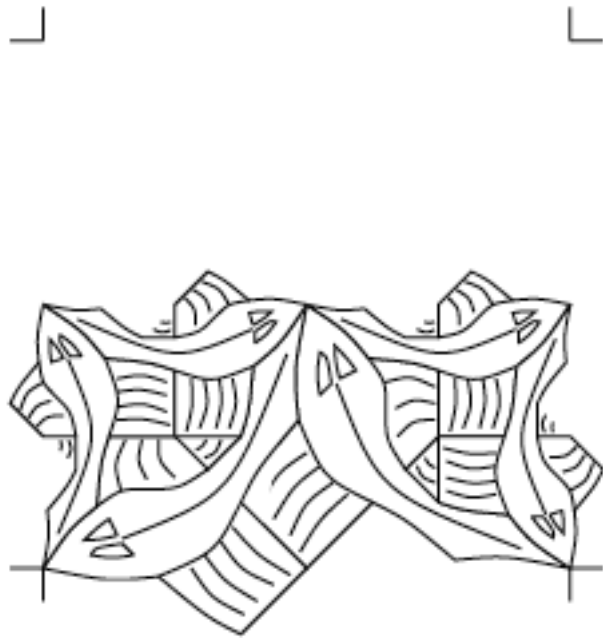


```
u = over (over (fish2, rot (fish2)),  
          over (rot (rot (fish2)),  
                rot (rot (rot (fish2))))))
```

P	Q
R	S

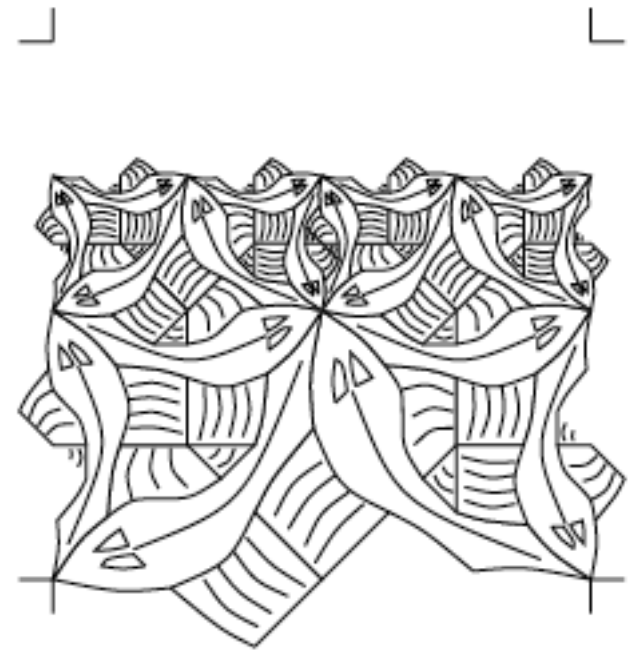
quartet





```
quartet(nil, nil,  
        rot(t), t)
```

**side1**

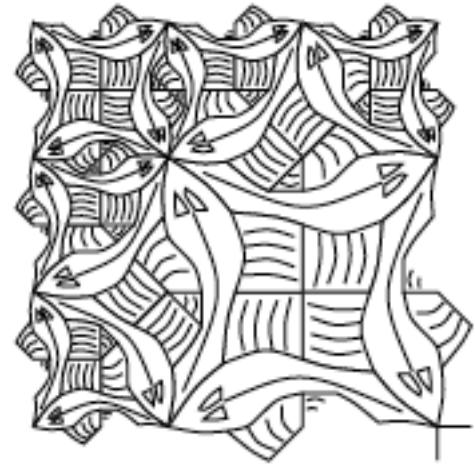


```
quartet(side1,side1,  
        rot(t), t )
```



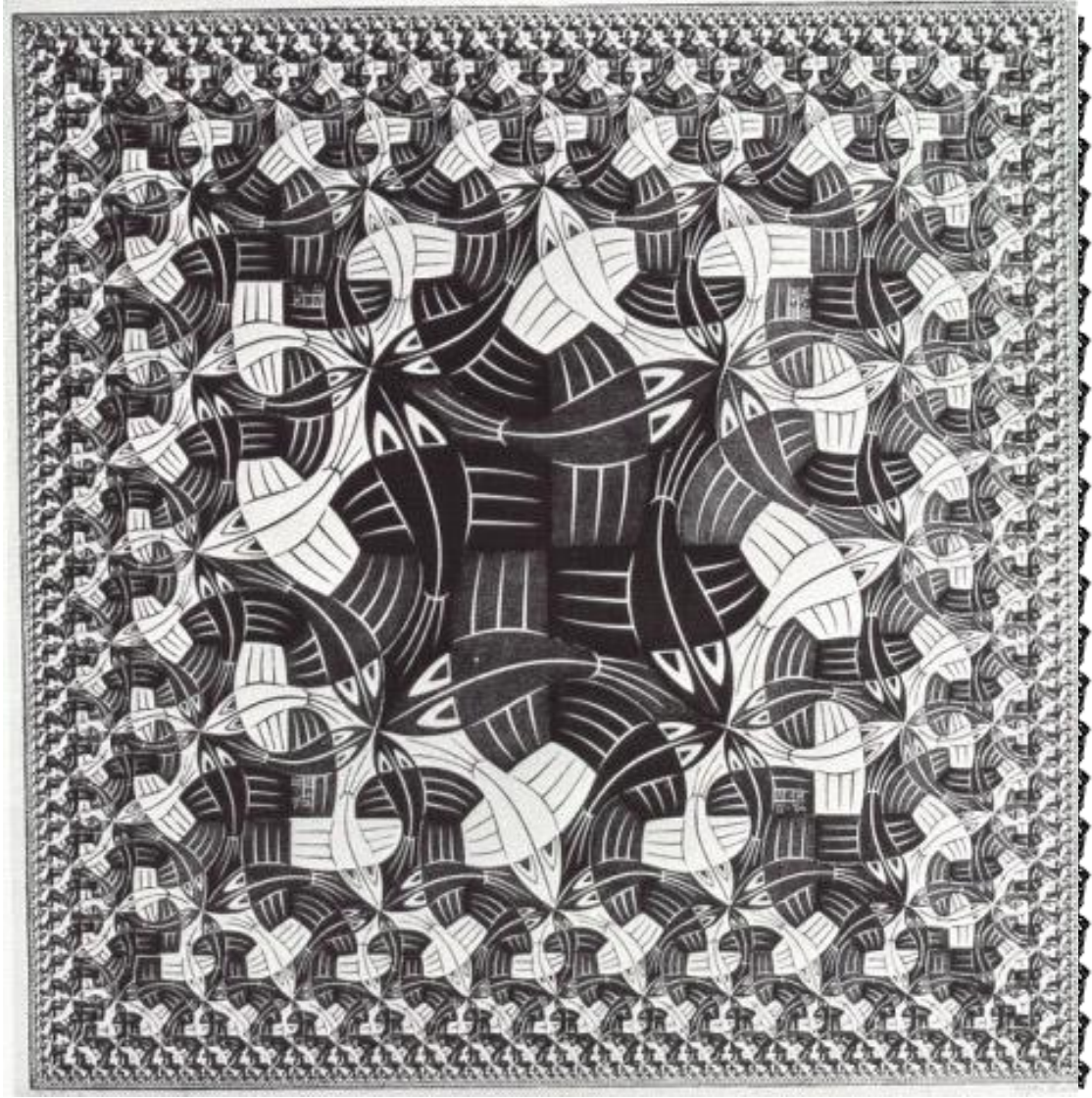
`quartet (nil,nil,nil,u)`

`corner1`



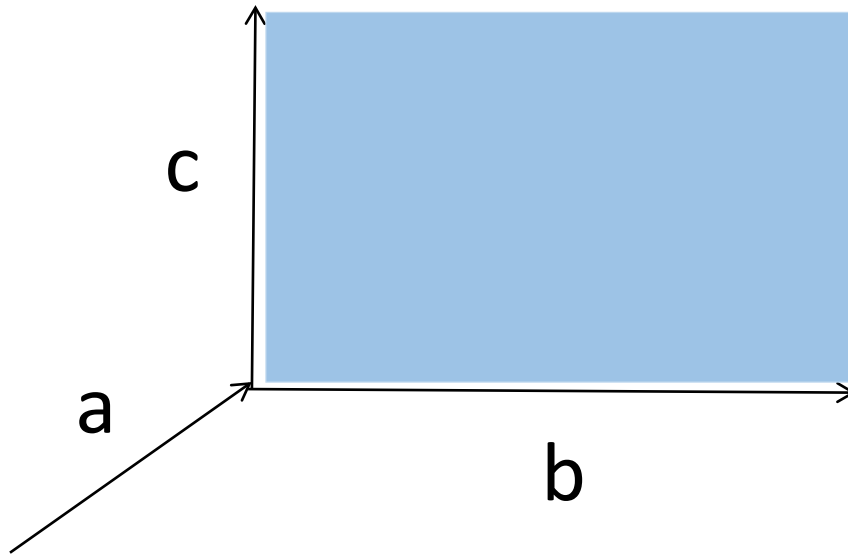
`quartet(corner1,  
side1,  
rot(side1),  
u)`

```
squarelimit = nonet(  
    corner,      side,      rot(rot(rot(corner))),  
    rot(side),  u,          rot(rot(rot(side))),  
    rot(corner), rot(rot(side)), rot(rot(corner)))
```

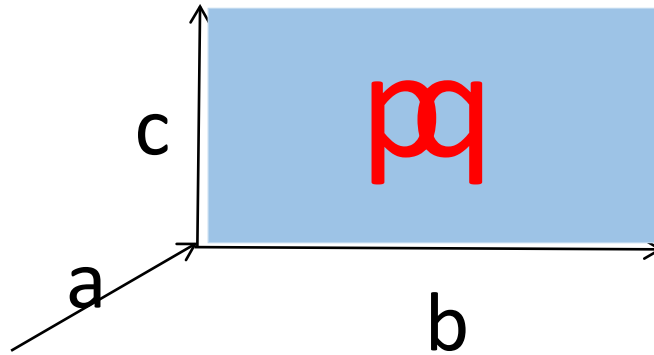


picture = function

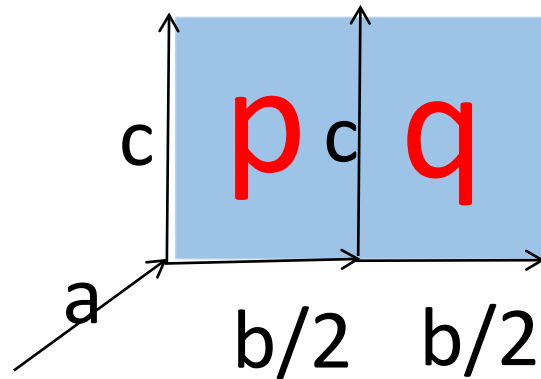
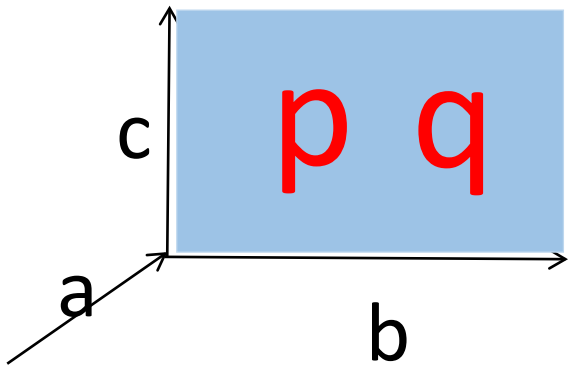
picture = function



$$\text{over } (p, q) (a, b, c) = p(a, b, c) \cup q(a, b, c)$$

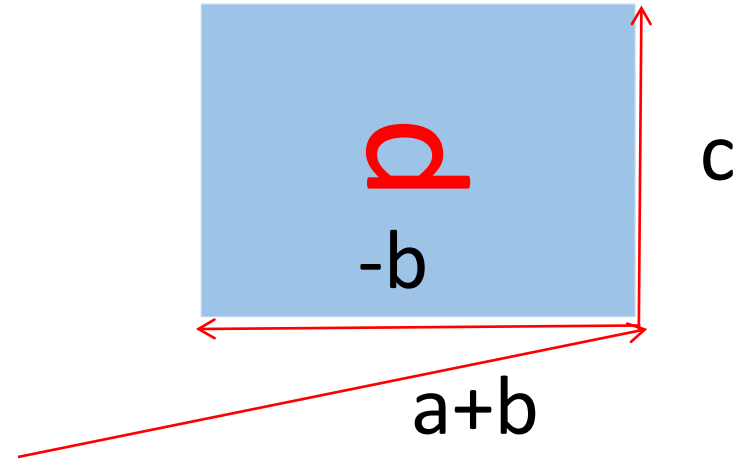
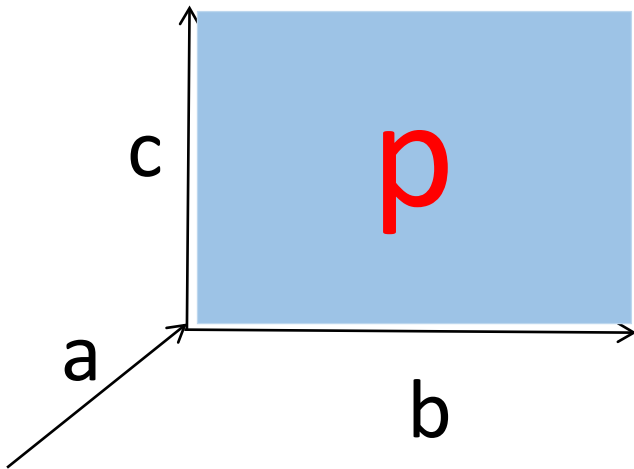


$$\text{beside } (p, q) \ (a, b, c) = p(a, b/2, c) \cup q(a+b/2, b/2, c)$$





$$\text{rot}(p) (a,b,c) = p(a+b,c,-b)$$



# Laws

$$\text{rot}(\text{above}(p, q)) \\ = \\ \text{beside}(\text{rot}(p), \text{rot}(q))$$



It seems there is a positive correlation between the simplicity of the rules and the quality of the algebra as a description tool.

Whole values

Combining forms

Algebra as litmus test

Functions as representations

# Haskell vs. Ada vs. C++ vs. Awk vs. ... An Experiment in Software Prototyping Productivity\*

Paul Hudak  
Mark P. Jones

Yale University  
Department of Computer Science  
New Haven, CT 06518

{hudak-paul, jones-mark}@cs.yale.edu



July 4, 1994



Time 40.0:

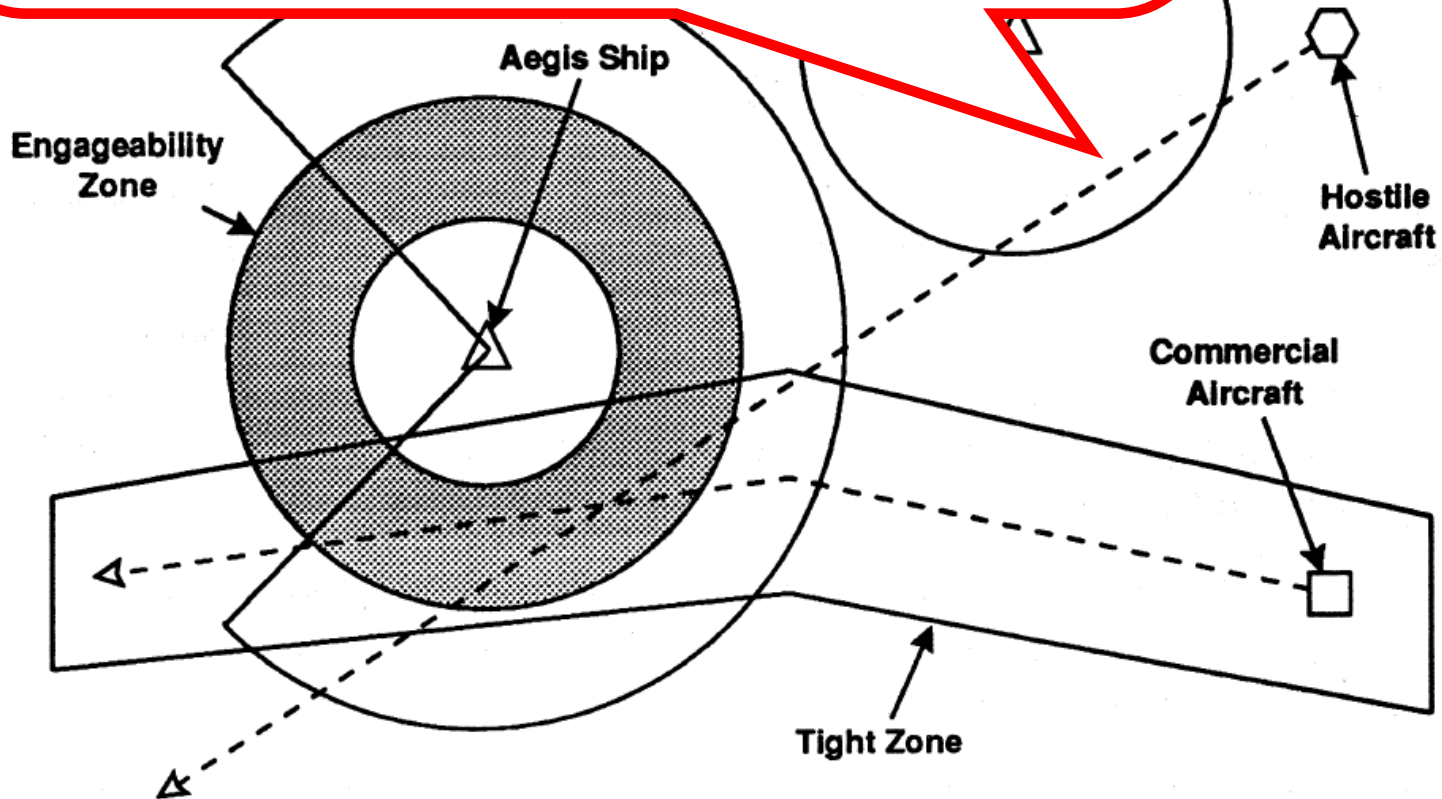
commercial aircraft: (100.0,43.0)

-- In engageability zone

-- In tight zone

hostile craft: (210.0,136.0)

-- In carrier slave doctrine



# Functions as Data

```
> type Region    = Point -> Bool

> circle        :: Radius -> Region
> outside       :: Region -> Region
> (/\)          :: Region -> Region -> Region

> annulus       :: Radius -> Radius -> Region
> annulus r1 r2 = outside (circle r1) /\ circle r2
```

Including 29 lines of inferable type signatures/synonyms

A student, given 8 days to learn Haskell, w/o knowledge of Yale group

Language	Lines of code	Lines of document	Development time (hours)
(1) Haskell	85	465	10
(2) Ada	767	7	23
(3) Ada9X	800	7	28
(4) C++	1105	130	-
(5) Awk/Nawk	250	150	-
(6) Rapide	157	0	54
(7) Griffin	251	0	34
(8) Proteus	293	79	26
(9) Relational Lisp	274	12	3
(10) Haskell	156	112	8

Figure 3: Summary of Prototype Software Development Metrics

# Reaction...

”too cute for its own good”

...higher-order functions just a trick, probably not useful in other contexts



# Lazy Evaluation (1976)



Henderson and Morris  
*A lazy evaluator*



Friedman and Wise  
*CONS should not evaluate  
its arguments*

# “The Whole Value” can be $\infty$ !

- The *infinite list* of natural numbers  
[0, 1, 2, 3 ...]

*Consumer* decides  
how many to  
compute

- All the iterations of a function

`iterate f x = [x, f x, f (f x), ...]`

- A consumer for numerical methods

`limit eps xs =`

*<first element of **xs** within **eps** of its predecessor>*

# Some numerical algorithms

- Newton-Raphson square root

```
sqrt a = limit eps (iterate next 1.0)  
  where next x = (x + a/x) / 2
```

- Derivatives

```
deriv f x =  
  limit eps (map slope (iterate (/ 2) 1.0))  
  where slope h = (f (x+h) - f x) / h
```

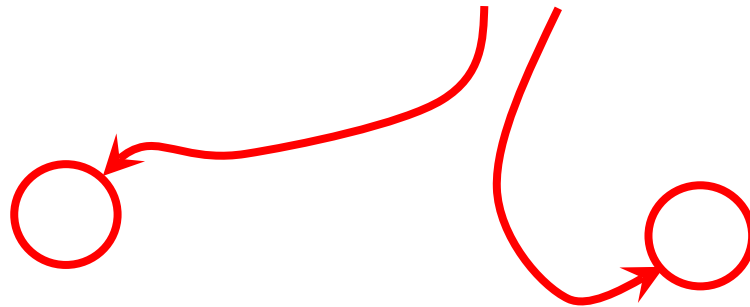
[1, 1/2, 1/4, 1/8...]

*Same convergence check*

*Different approximation sequences*

# Speeding up convergence

The smaller  $h$  is, the better the approximation



Differentiation

Integration

The right answer

$$A + B * h^n$$

An error term

# Eliminating the error term

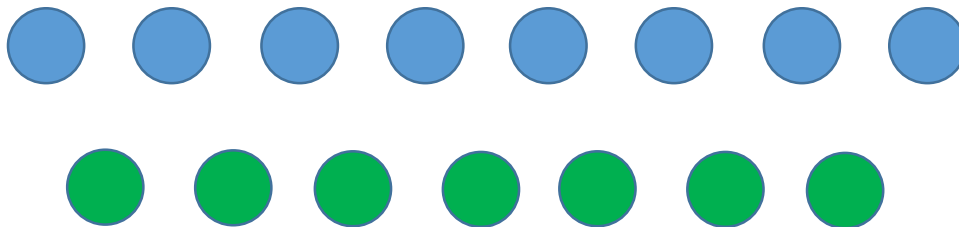
- Given:

$$A + B * h^n$$

$$A + B * (h/2)^n$$

Two successive approximations

- Solve for A and B!



**improve n xs converges faster than xs**

# Really fast derivative

```
deriv f x =  
  limit eps  
    (improve 2  
      (improve 1  
        (map slope (iterate (/2) 1.0))))
```

*The convergence check*

*The improvements*

*The approximations*

Everything is programmed *separately* and easy to understand—thanks to “whole value programming”

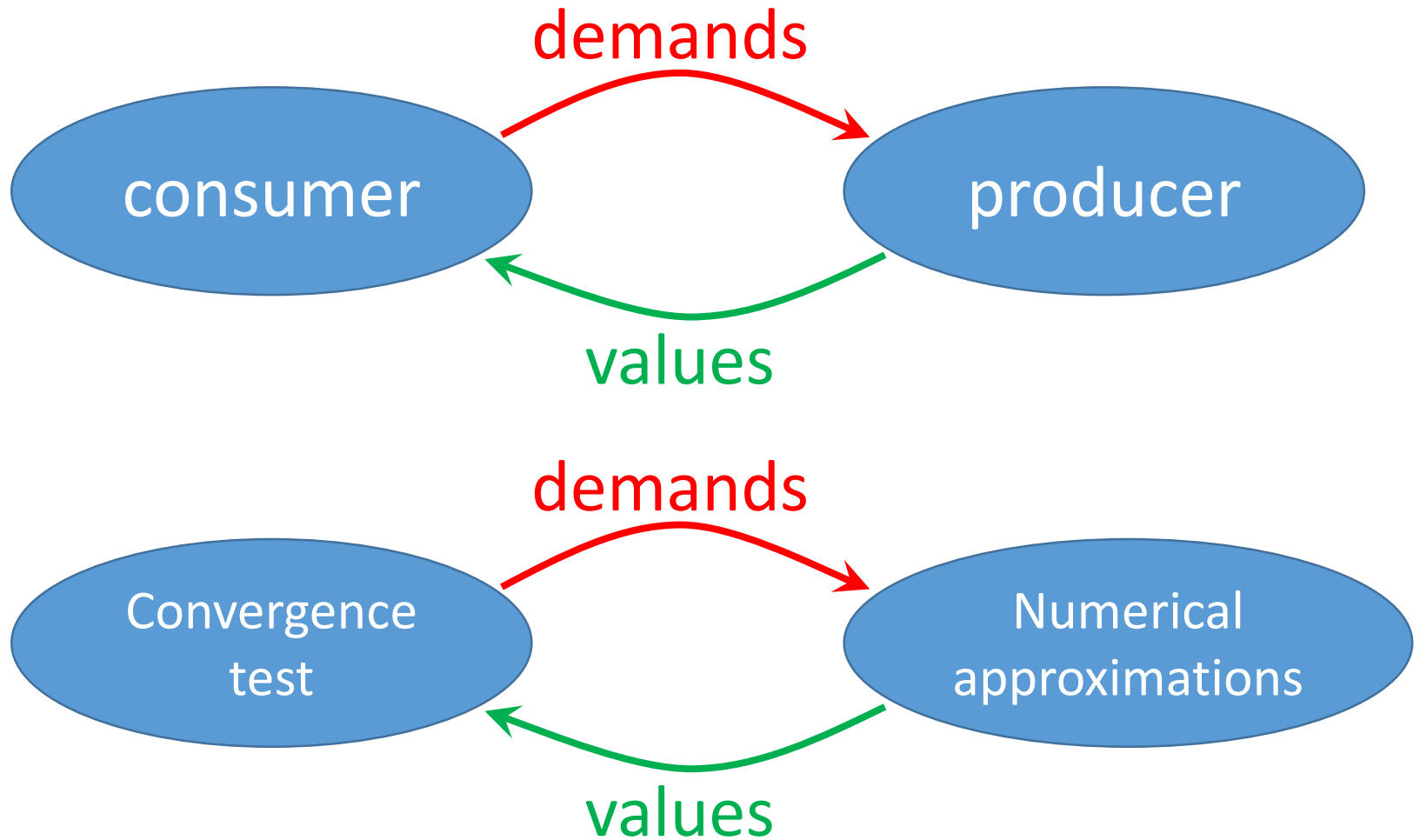
# Why Functional Programming Matters

John Hughes  
The University, Glasgow



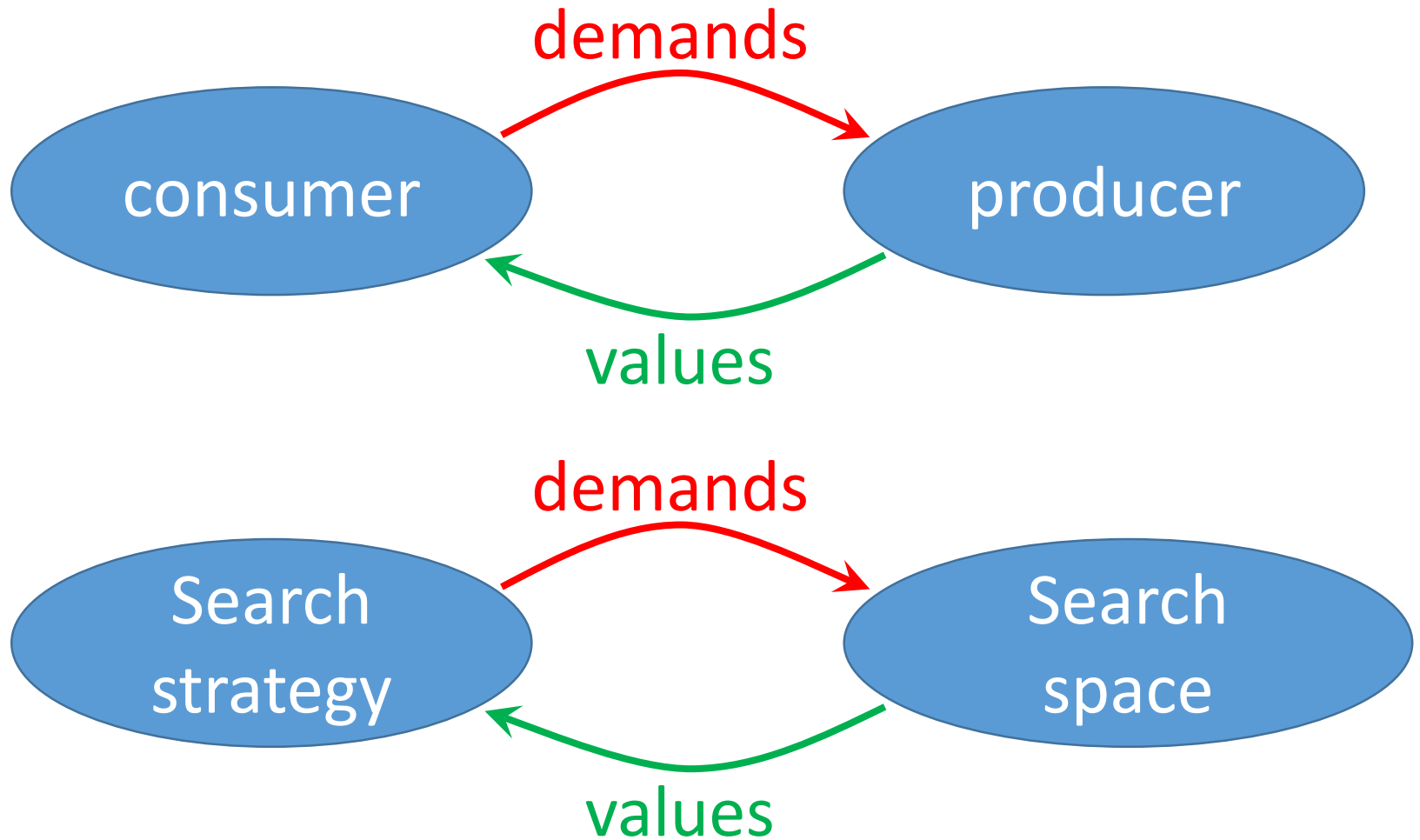
1990

# Lazy producer-consumer





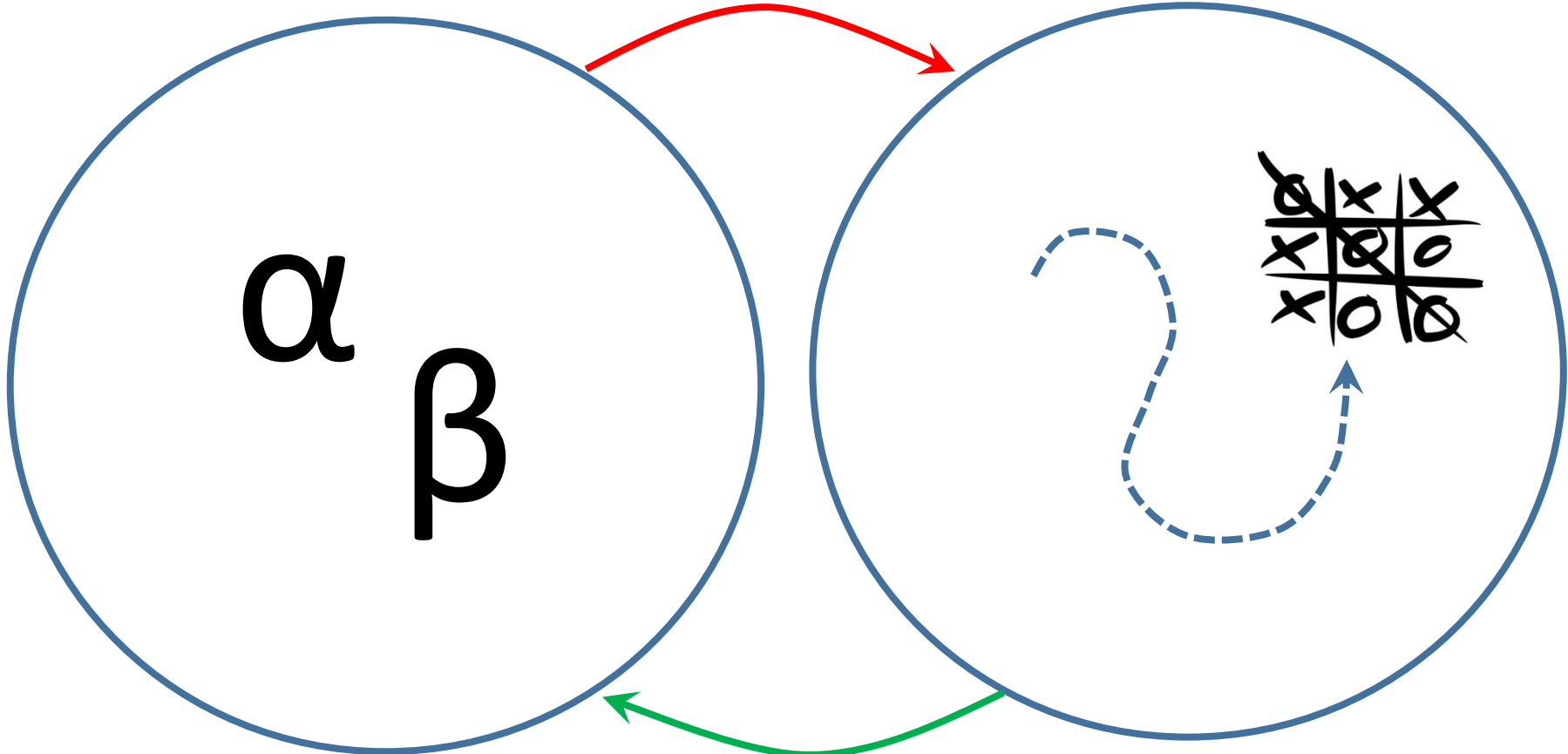
# Lazy producer-consumer



# Why Functional Programming Matters

John Hughes  
The University, Glasgow

demands



values

# The Design of a Pretty-printing Library

John Hughes

Chalmers Tekniska Högskola, Göteborg, Sweden.



demands



Selection  
criterion for  
the best  
layout

A document  
with many  
alternative  
layouts



values

# Combinators!

## **Ways to combine "documents with many layouts"**

- Horizontal composition
- Vertical composition
- Indentation
- Choice (horizontal/vertical)

(Branch

(Branch (Leaf 1) (Leaf 2))

(Branch

(Leaf 3)

(Branch (Leaf 4) (Leaf 5))))

```
pretty (Leaf a) =  
  text ("(Leaf "++show a++)")
```

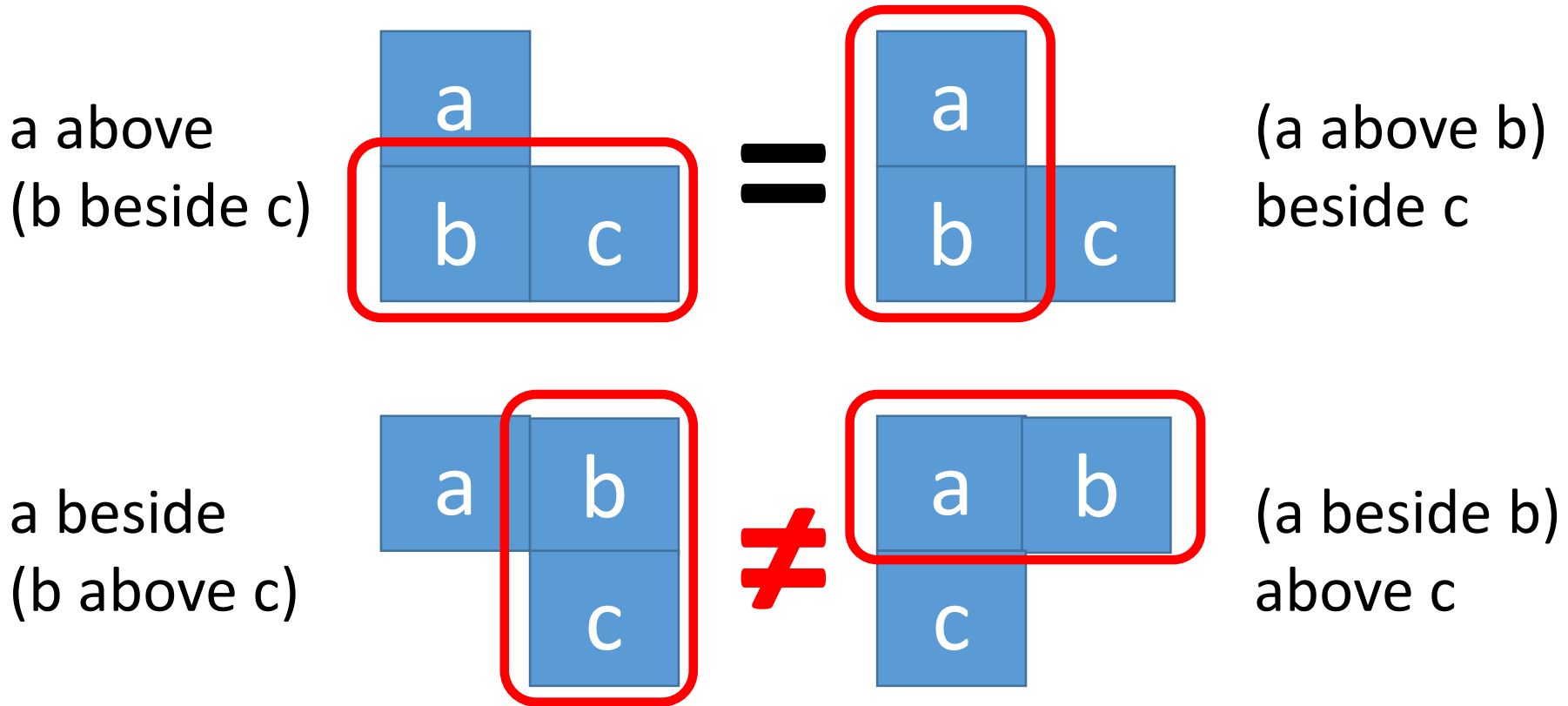
*Fixed layout*

```
pretty (Branch l r) =  
  sep [text "(Branch",  
       nest 2 (pretty l),  
       nest 2 (pretty r)<>text ")"]
```

*Choice*

*Indent*

# Laws!



***Algebra as a litmus test!***

***The laws drive the implementation***

# How do I find the laws?



*QuickSpec: guessing formal specifications using testing.* Claessen, Smallbone and Hughes. TAP 2010.

*Quick specifications for the busy programmer.*  
Smallbone, Johansson, Claessen and Algehed. 2017.

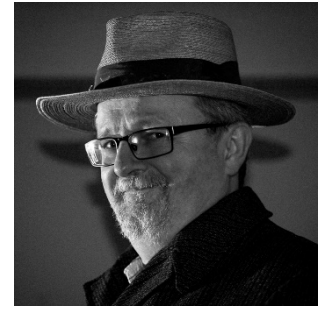
# Improved on by...







# QuickCheck: A Lightweight Tool for Random Testing of Haskell Programs



Koen Claessen  
Chalmers University of Technology  
koen@cs.chalmers.se

John Hughes  
Chalmers University of Technology  
rjmh@cs.chalmers.se

```
prop_reverse() ->
  ?FORALL(Xs, list(int()),
    reverse(reverse(Xs)) == Xs).
```

```
3> eqc:quickcheck(qc:prop_reverse()).
```

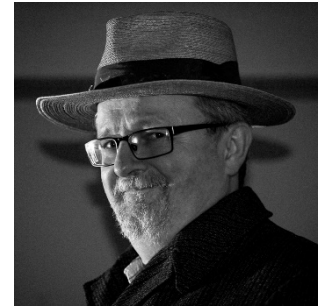
```
.....
.....
```

```
OK, passed 100 tests
```

```
true
```



# QuickCheck: A Lightweight Tool for Random Testing of Haskell Programs



Koen Claessen  
Chalmers University of Technology  
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Chalmers University of Technology  
rjmh@cs.chalmers.se

```
prop_wrong() ->  
  ?FORALL(Xs, list(int()),  
          reverse(Xs) == Xs).
```

```
4> eqc:quickcheck(qc:prop_wrong()).
```

```
Failed! After 1 tests.
```

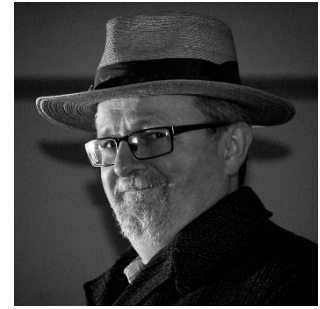
```
[-36,-29,20,31,-47,-63,80,-7,93,-87,-29,33,64,58]
```

```
Shrinking xx.x.x..xx(4 times)
```

```
[0,1] ← minimal counterexample  
false
```

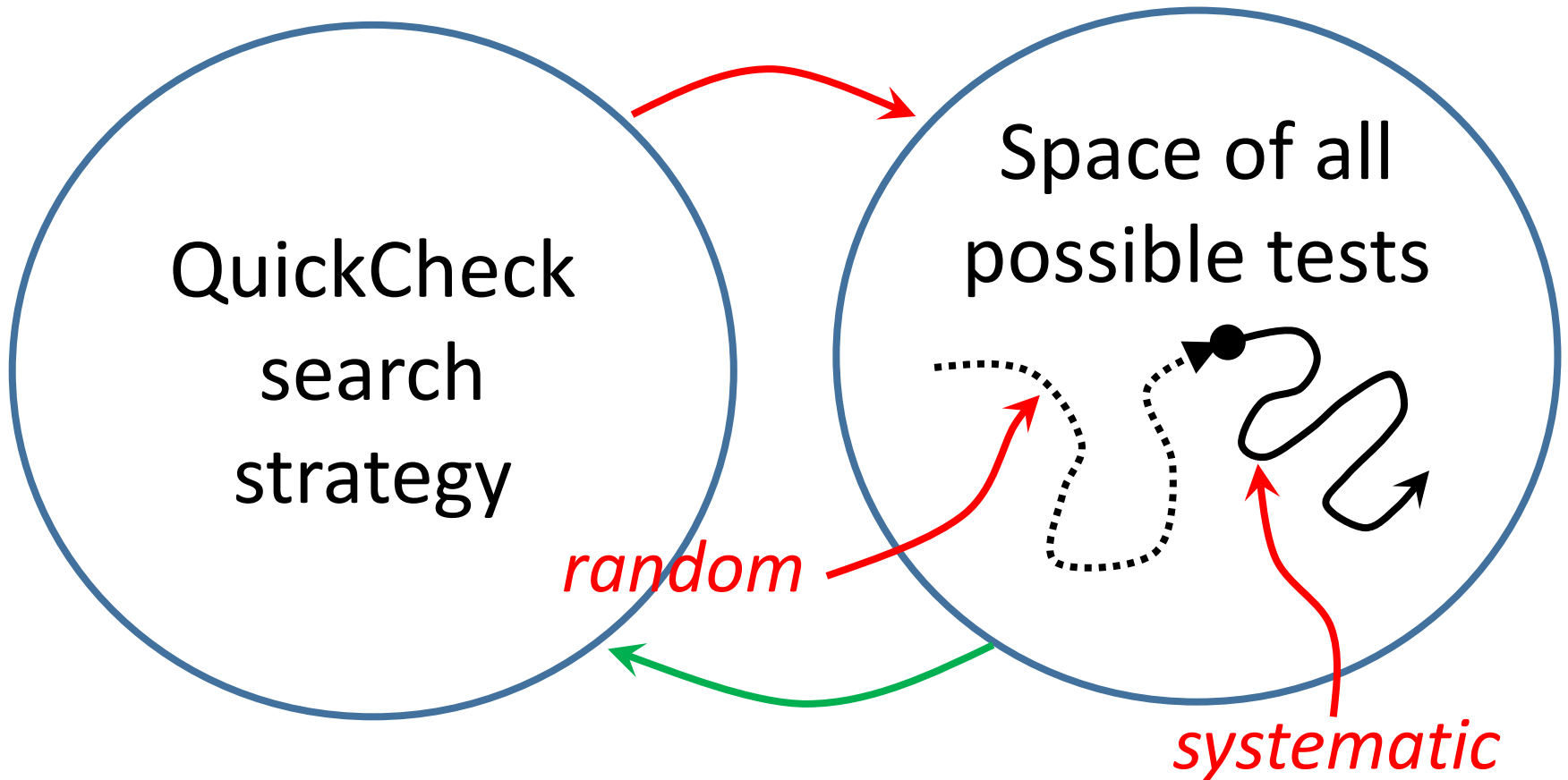


# QuickCheck: A Lightweight Tool for Random Testing of Haskell Programs



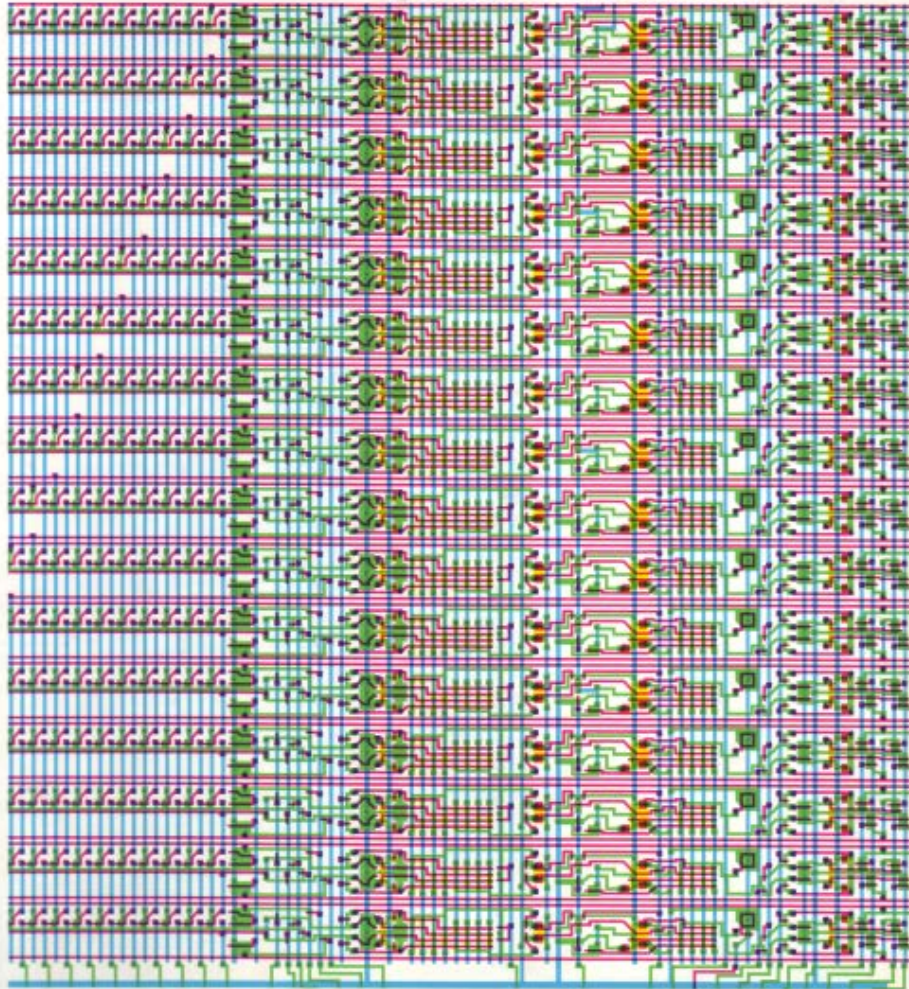
Koen Claessen  
Chalmers University of Technology  
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rjmh@cs.chalmers.se



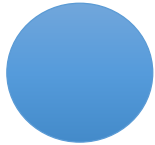
# INTRODUCTION TO **VLSI** SYSTEMS

CARVER MEAD • LYNN CONWAY

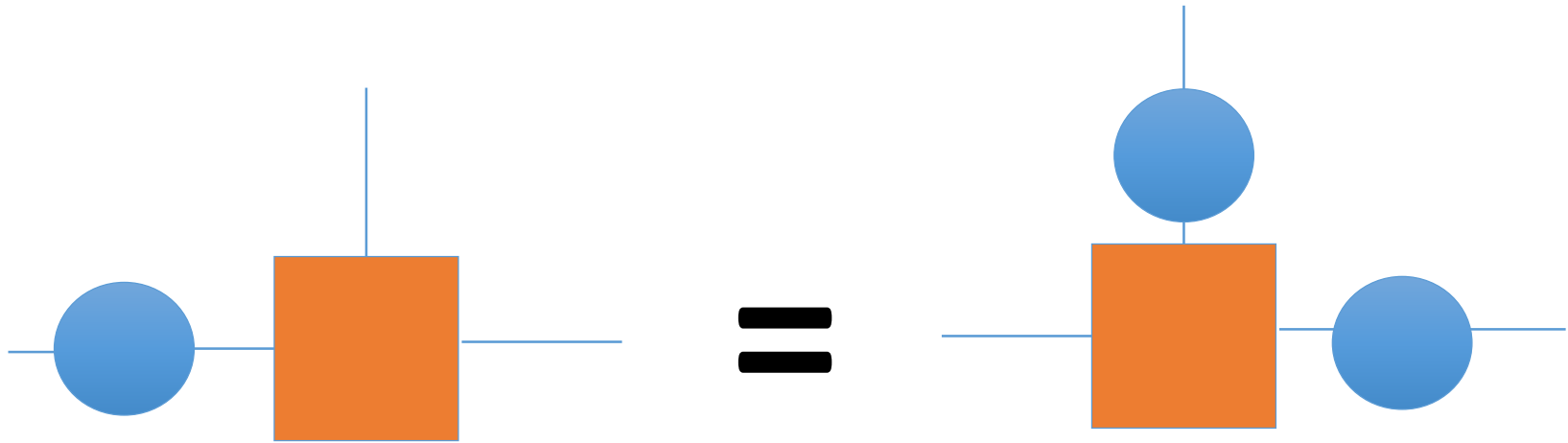


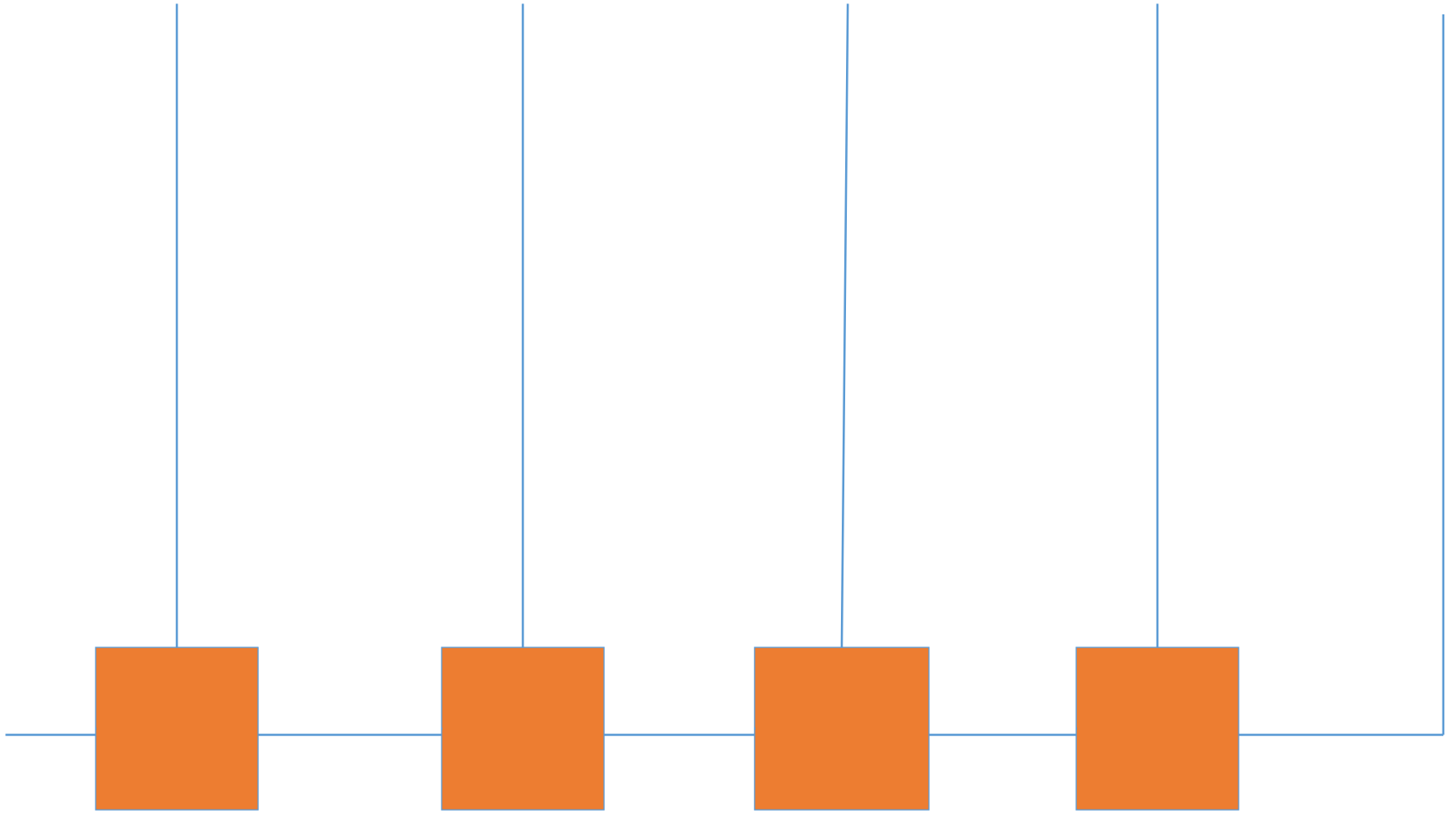
# muFP—Circuits as values

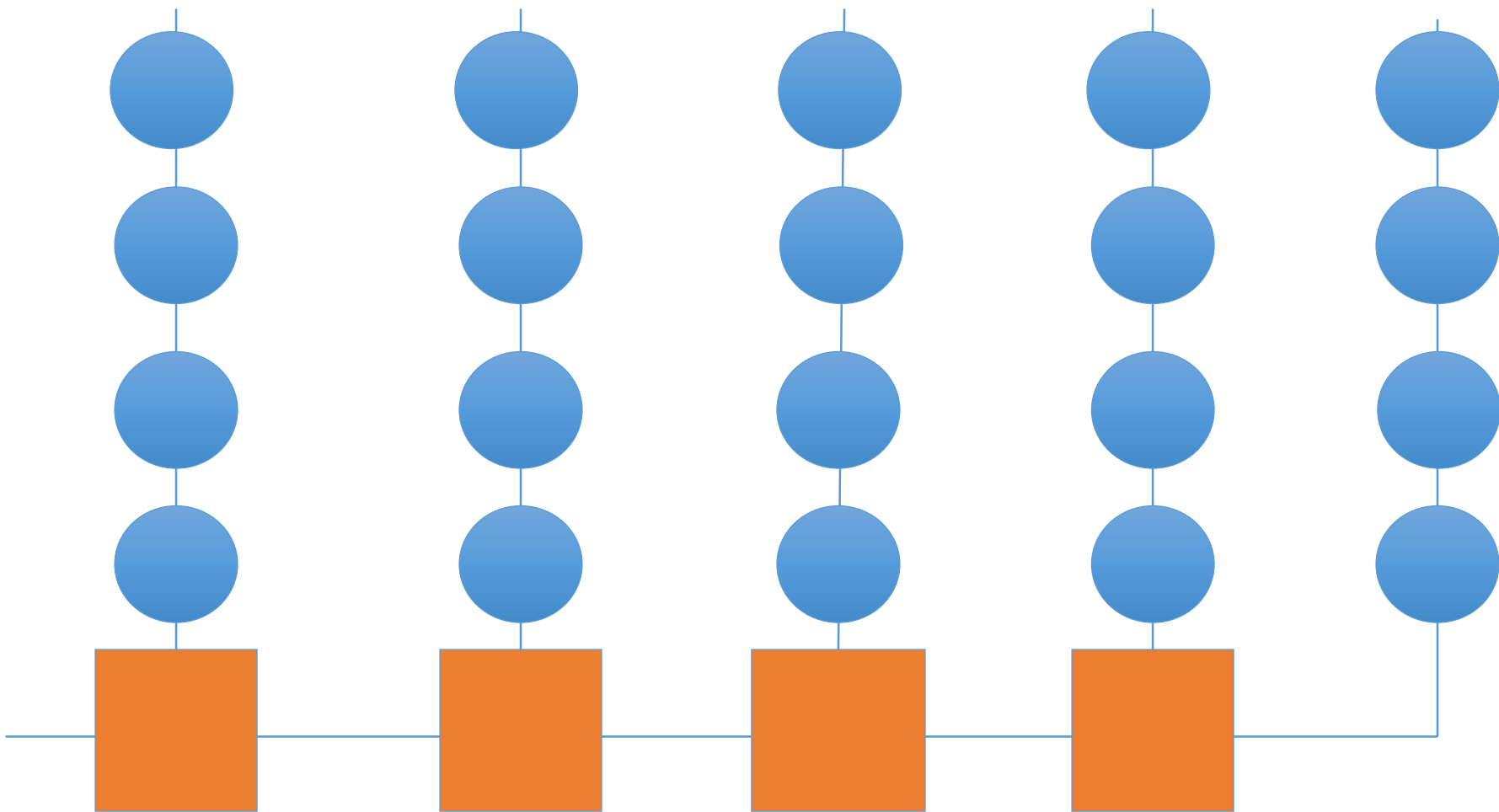


- Backus FP + unit delays 
- Inherits many combining forms and laws
- Good for reasoning about alternative designs

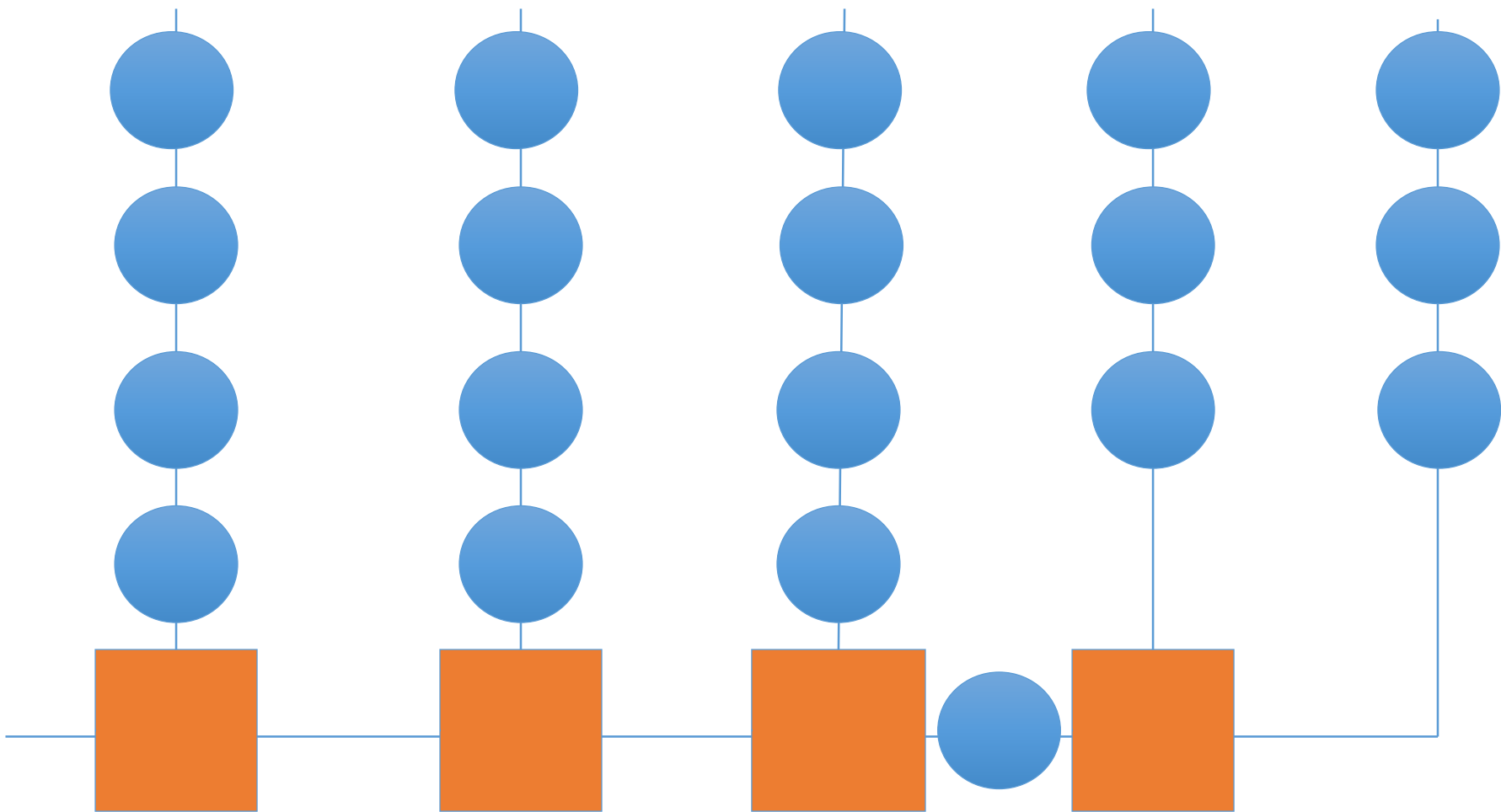
# Example law: “retiming”

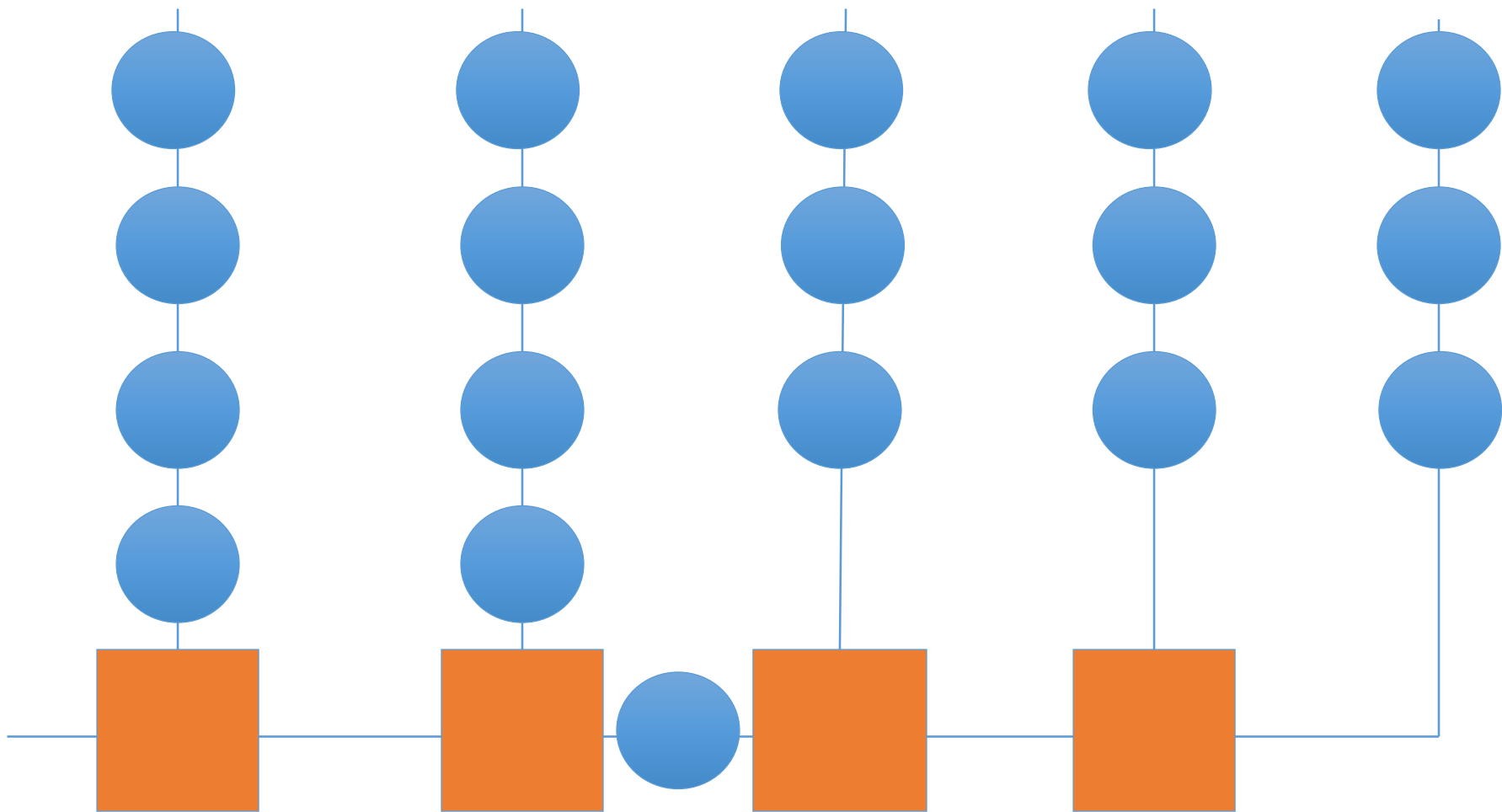


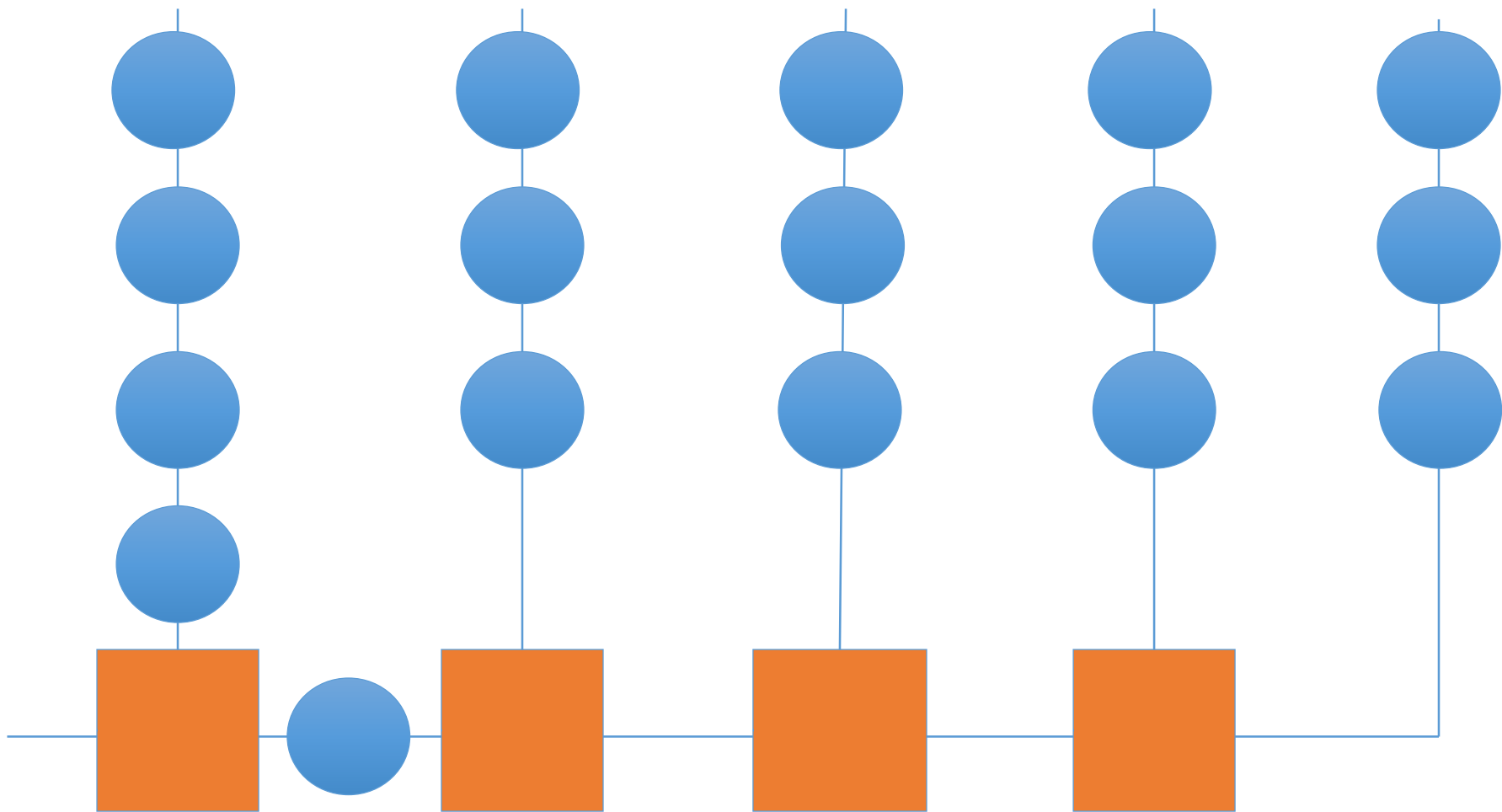


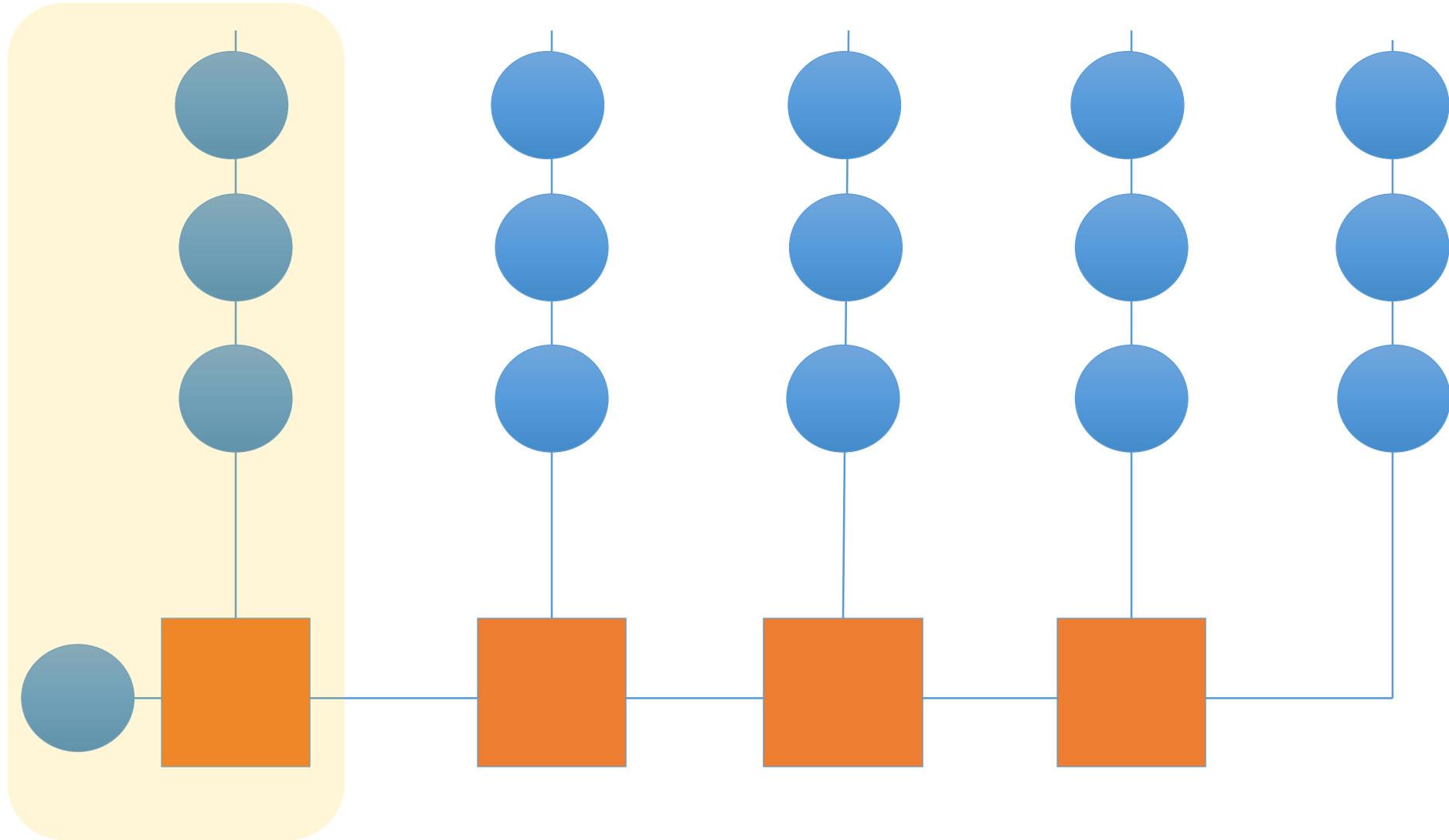


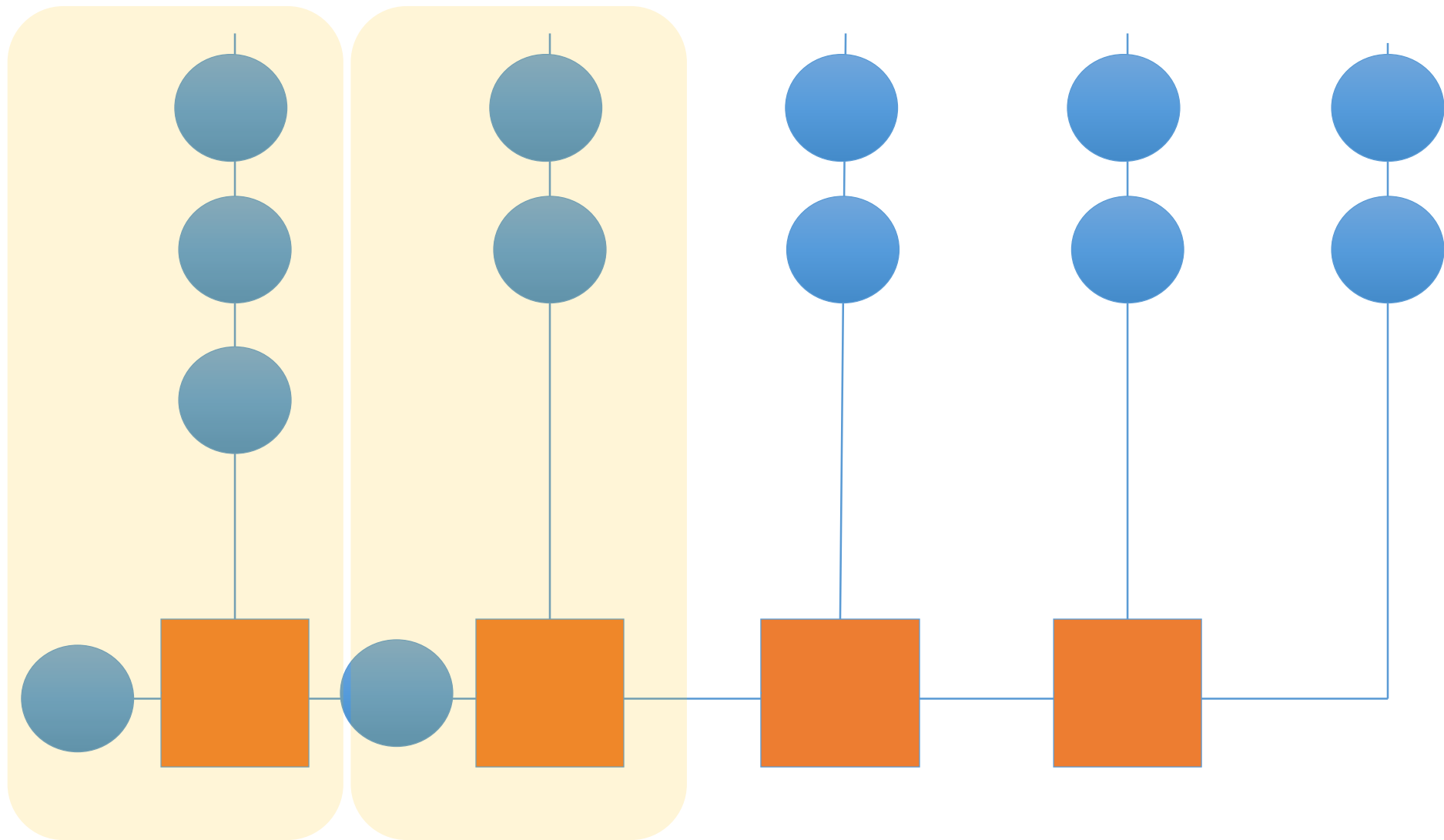


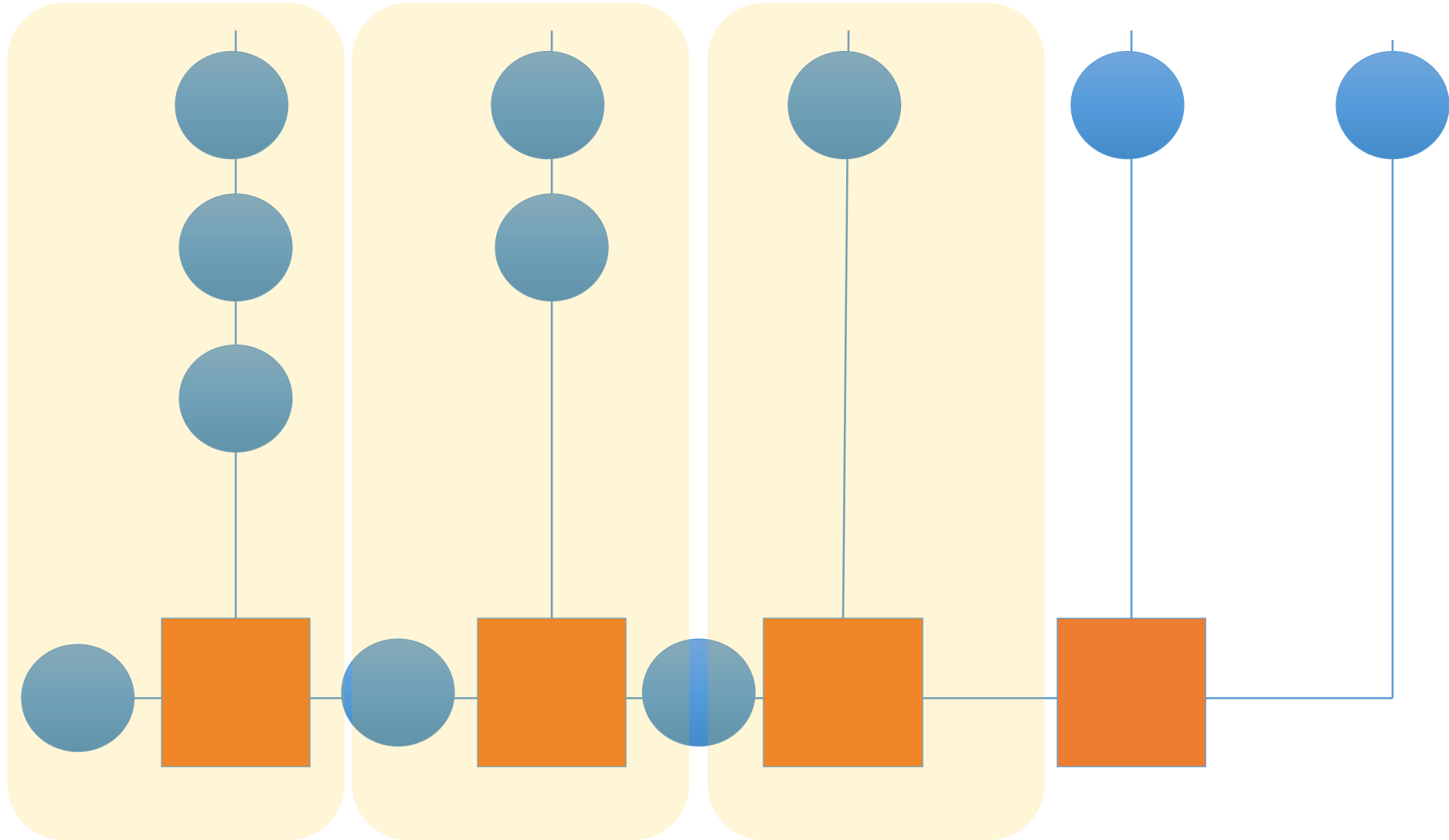


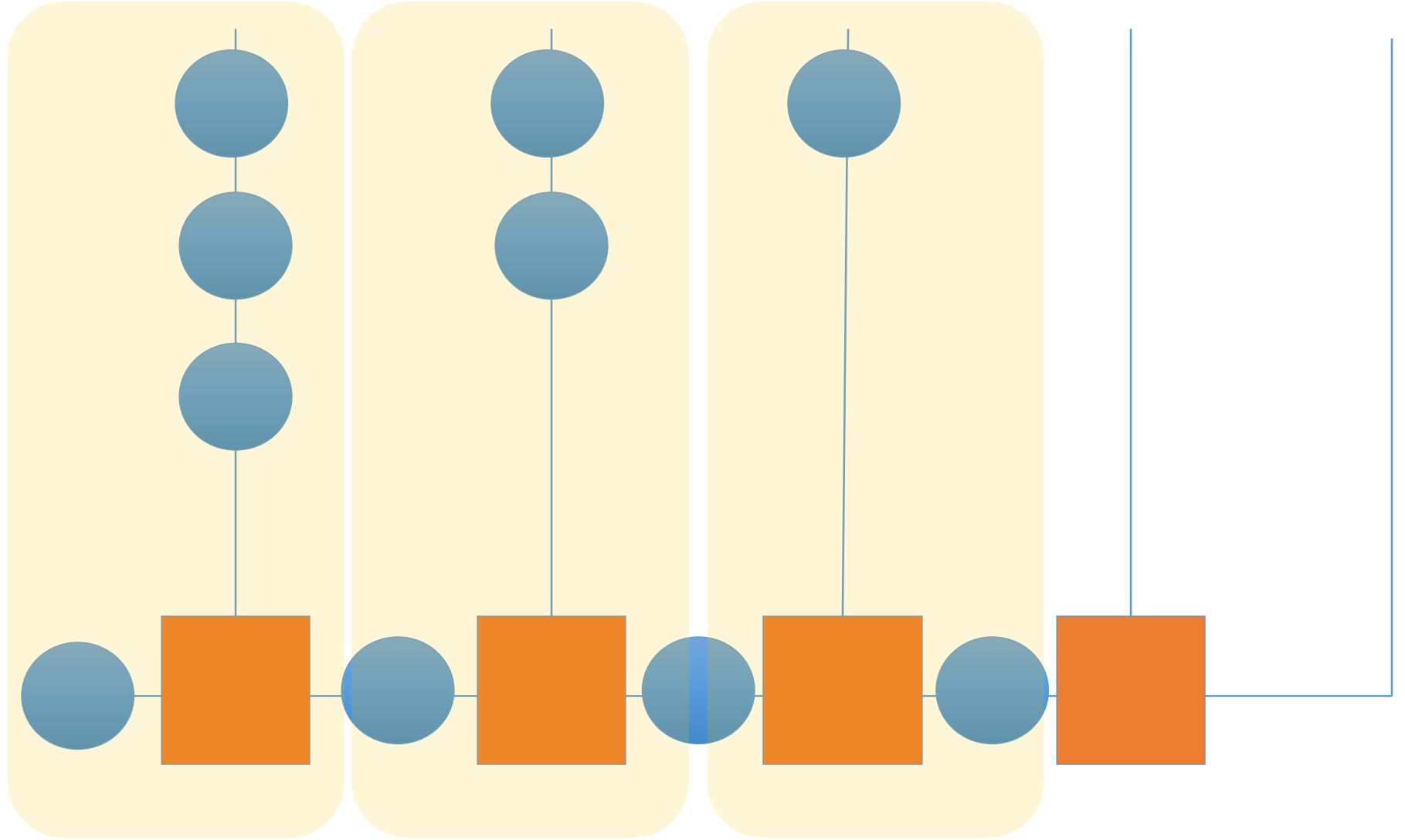












# Users!

## Plessey video motion estimation

“Using muFP, the array processing element was described in just one line of code and the complete array required four lines of muFP description. muFP enabled the effects of adding or moving data latches within the array to be assessed quickly.”

Bhandal et al, An array processor for video picture motion estimation, Systolic Array Processors, 1990, Prentice Hall

work with Plessey done by G. Jones and W. Luk



# Lava

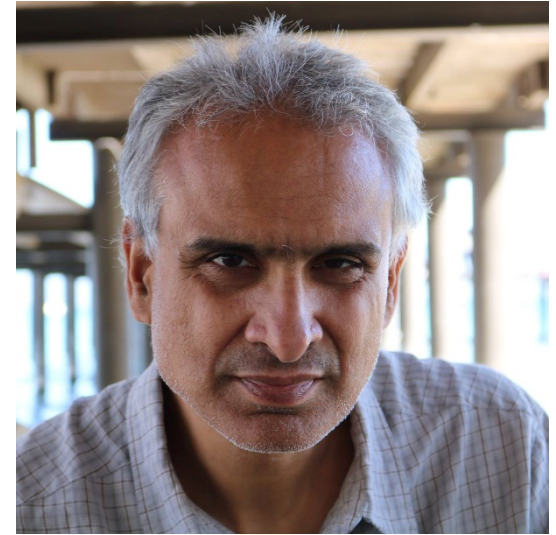
*Semantics*

muFP

+

Functional  
Geometry

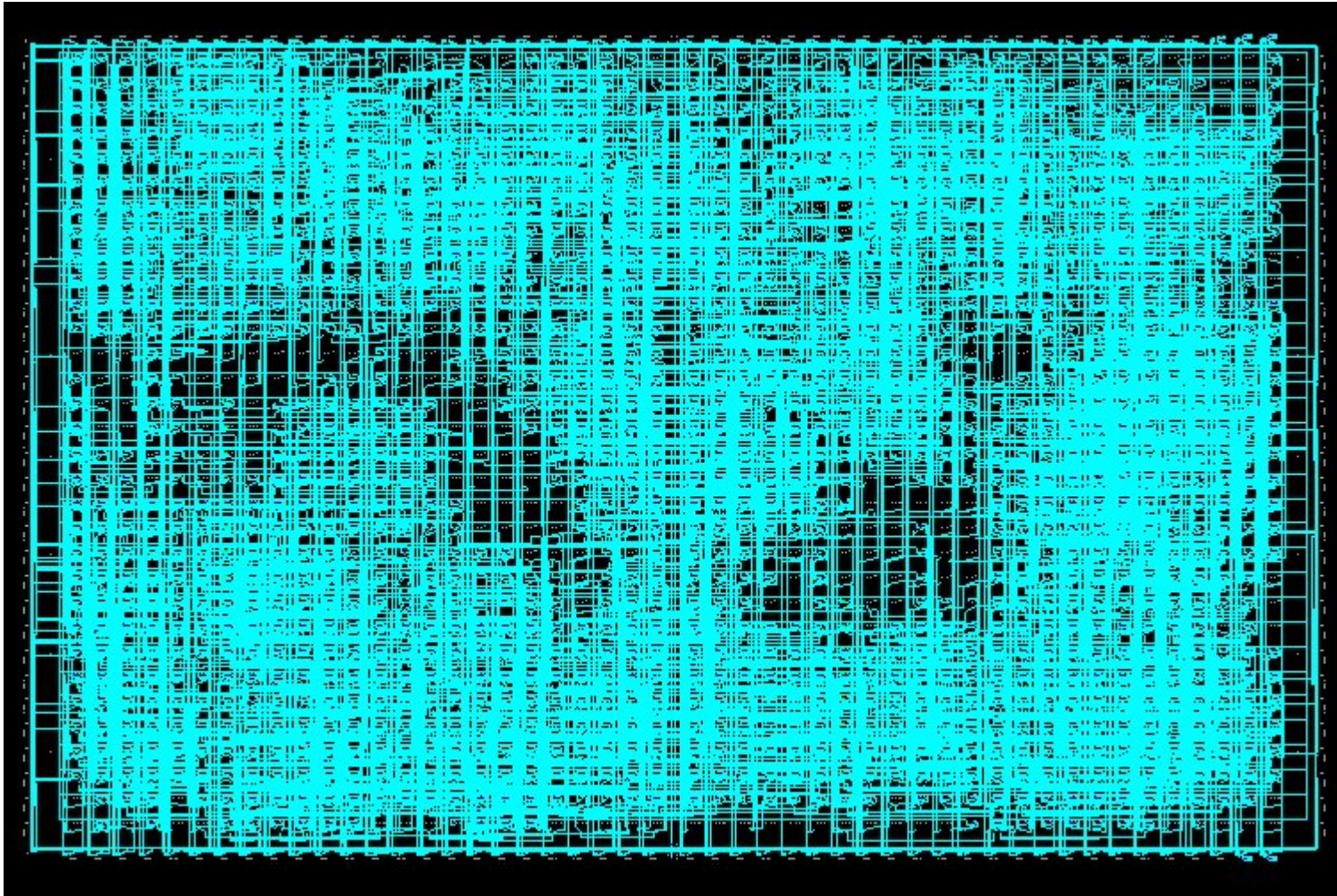
*Placement*



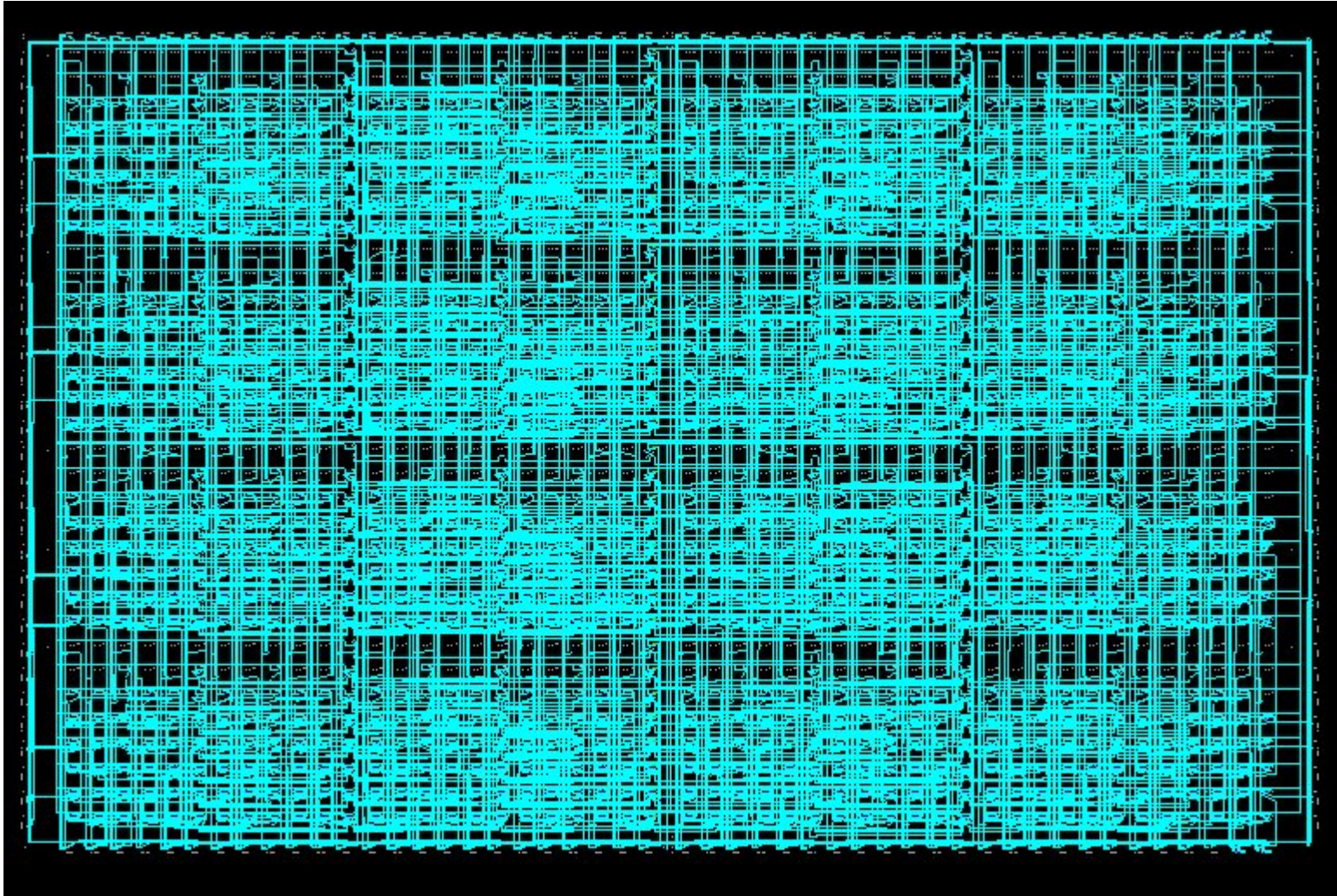
Satnam Singh  
Xilinx

➔ FPGA layouts on Xilinx chips

# Four adder trees—no placement



# Four adder trees—Lava



# Intel

$$4195835.0 - 3145727.0 * (4195835.0 / 3145727.0) = 0$$

# Intel

$$4195835.0 - 3145727.0 * (4195835.0 / 3145727.0) = 0$$

Flawed Pentium

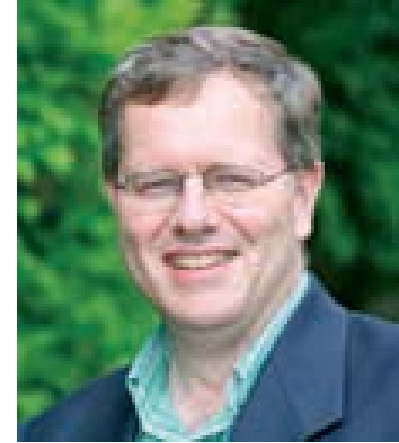
$$4195835.0 - 3145727.0 * (4195835.0 / 3145727.0) = 256$$

# \$475 million



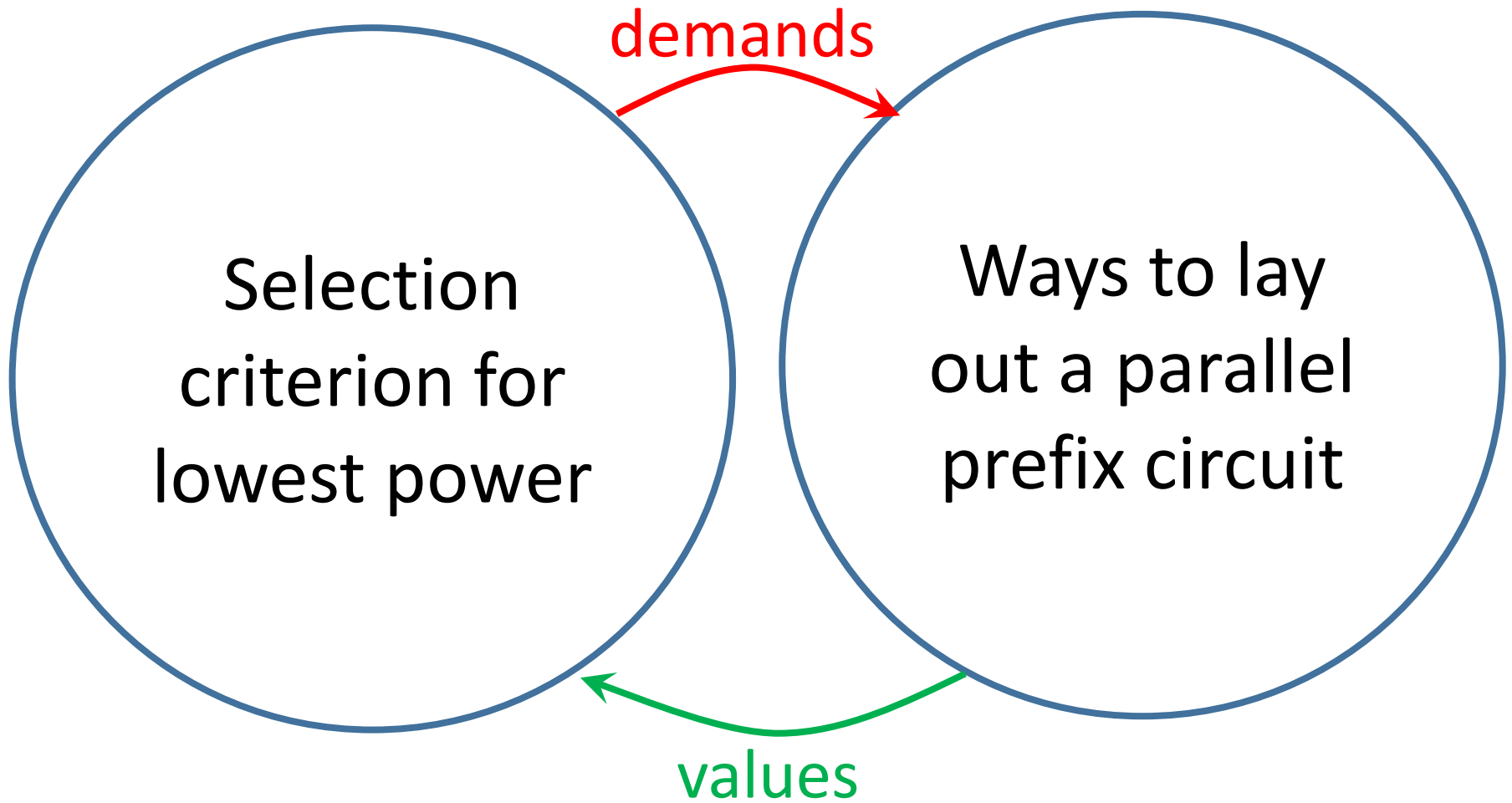
# fl

Lazy functional language,  
1000s users



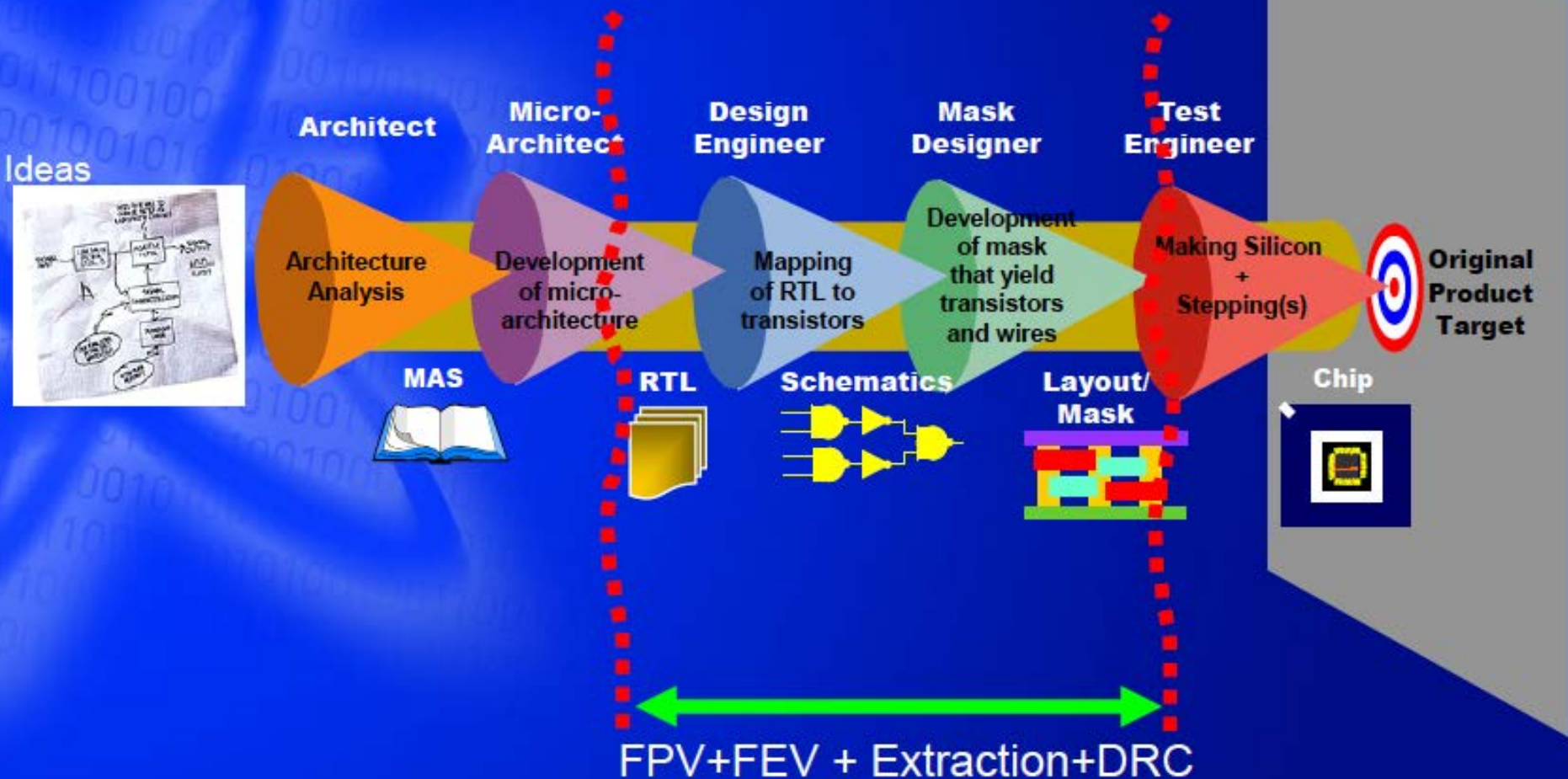
- Design
- High-level specification
- Scripting
- Implementation of formal verification tools and theorem provers
- Object language for theorem proving

Thanks to Carl Seger (formerly of Intel)





# Solid Formal Link with Good Return of the Investment



Thanks to Carl Seger (formerly of Intel)

# Bluespec—FP for hardware



Haskell-like language (architecture) +  
atomic transition rules (H/W modelling)

Frees designers to explore *better algorithms*,  
making major architectural change easy



Types, Functional Programming and  
Atomic Transactions in Hardware Design  
Rishiyur Nikhil LNCS 8000

# Bluecheck

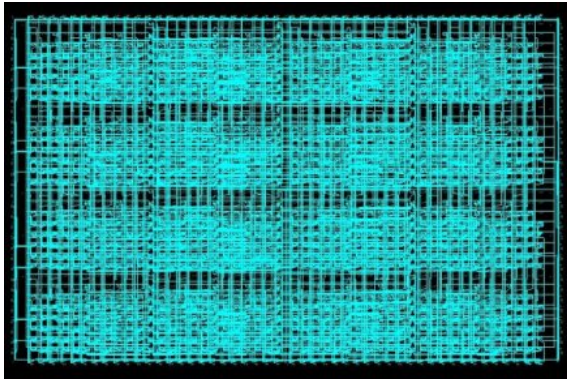
- QuickCheck in Bluespec!
- Generates and shrinks tests *on the FPGA!*

[A Generic Synthesisable Test Bench \(Naylor and Moore, Memocode 2015\)](#)

two  $f\ x = f\ (f\ x)$

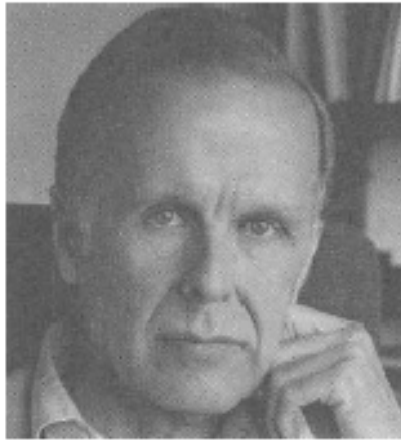
one  $f\ x = f\ x$

zero  $f\ x = x$



**Whole  
values**

**Combining  
forms**



**Simple  
laws**

**Functions as  
representations**